MECE 3350U
Control Systems

Lecture 23

Final Examination Review and Practice Exercises

Gain and phase - review

For a generic transfer function $G(s)$

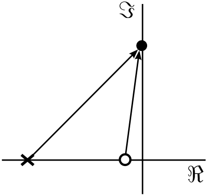
$$G(s) = k\frac{\prod_{i=1}^{n}(s+z_i)}{\prod_{k=1}^{m}(s+p_k)}$$

we can evaluate the **phase** at a frequency ω by letting $s = j\omega$.

The phase is

$$\angle G(j\omega) = \angle|k| + \sum_{i=1}^{n}\angle(j\omega + z_i) - \sum_{k=1}^{m}\angle(j\omega + p_k)$$

where $\angle(j\omega + a) = \tan^{-1}\omega/a$



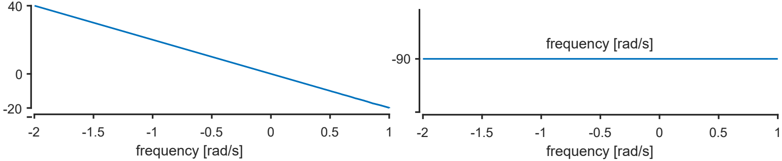
Bode plot building blocks

1 - Constant gain

→ Gain: $|k|$ or $20\log(|k|)$

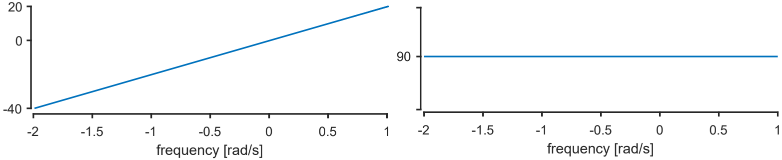
→ Phase: $\phi = 0\ \forall\ \omega$ if $k > 0$, -180° otherwise

2 - Pole at the origin

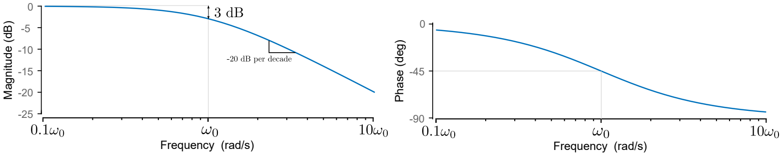


Bode plot building blocks

3 - Zero at the origin

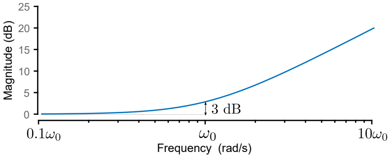


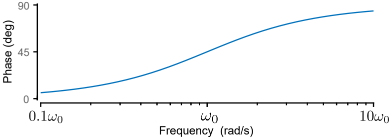
4 - Real pole: $G(s) = \frac{1}{\frac{s}{\omega_0}+1}$, $\omega_0 \in \Re^*$



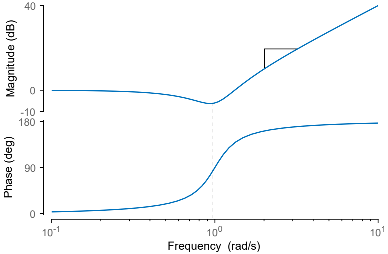
Bode plot building blocks

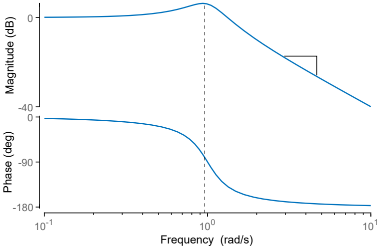
5 - Real zero: $G(s) = \frac{s}{\omega_0} + 1,\ \omega_0 \in \Re^*$



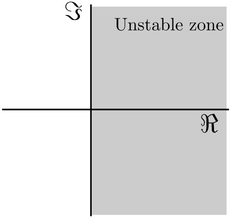


6 - Imaginary zeros or poles:





Open loop *vs* closed loop stability



Open-loop stability

$$T(s) = C(s)G(s)$$

$\rightarrow$ Evaluate the location of the **poles** of $C(s)G(s)$

Closed-loop stability

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

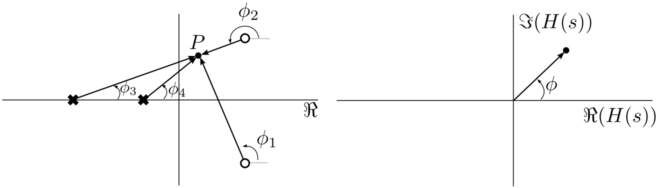
$\rightarrow$ Evaluate the location of the **zeros** of $1 + C(s)G(s)$

Example: If $C(s)G(s) = \frac{s+a}{s+b}$

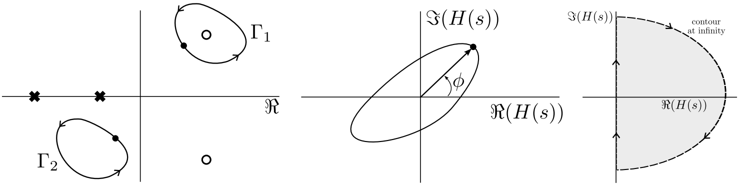
$\rightarrow$ Open-loop stable if $C(s)G(s)$ has real negative **poles**: i.e., $b > 0$

$\rightarrow$ Closed-loop stable if $1 + C(s)G(s)$ has real negative **zeros**:

Cauchy's argument principle



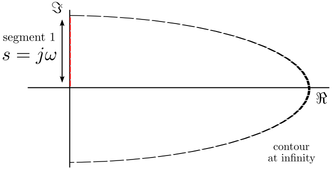
A contour map of a complex function will encircle the origin $N = Z - P$ times, where Z is the number of zeros and P is the number of poles of the function inside the contour.



The Nyquist Stability Criterion

A open-loop transfer function $L(s)$ is closed-loop stable if and only if the number of counterclockwise encirclements of the $-1+0j$ point is equal to the number of poles of $L(s)$ with positive real parts

$$Z = N + P$$



Nyquist plot

→ The contour at infinity maps to a single point

→ $\omega = 0$ (starting point)

→ $\omega \to \infty$

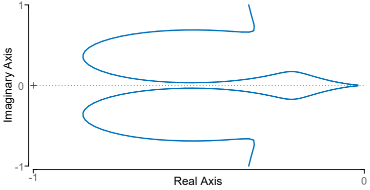
→ Point where the plot crosses the real and imaginary axis

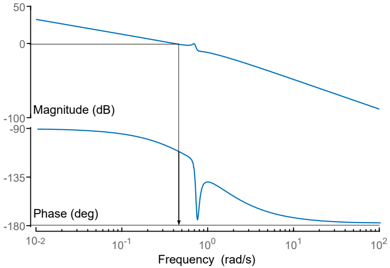
Gain and phase margins

The characteristic equation of a closed loop system with unit feedback is

$$1 + C(s)G(s) = 0$$

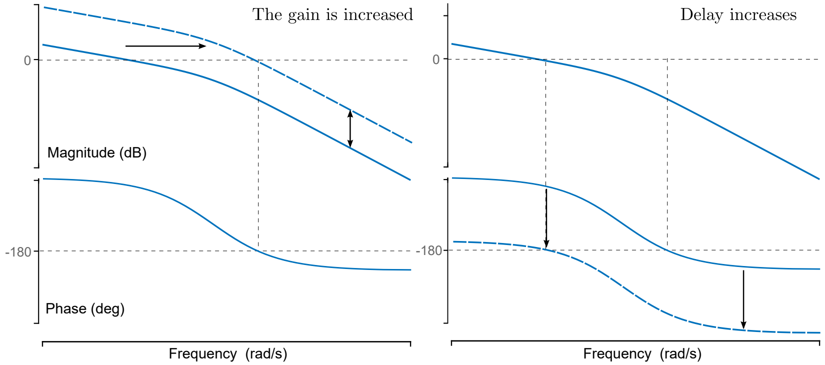
If $|C(s)G(s)| = 1$ and $\angle|C(s)G(s)| = \pm 180^\circ$, the characteristic equations is zero





Stability margin: How far the system if from $-1 + 0j$ or $1\angle 180^\circ$

Gain and phase margins



Phase and gain margin

Phase margin

Step 1 - Find the crossover frequency (0 dB). At the crossover frequency $\omega = \omega_c$, the magnitude is 1

Step 2 - Find the phase of $G(j\omega)$ at ω_c for ω_c found in Step 1, i.e. $\angle G(j\omega_c)$

Step 3 - The margin phase is $180 - |\phi|$

Gain margin

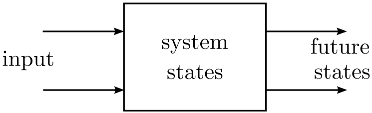
Step 1 - Find the frequency ω_f where $\angle|G(j\omega)| = -180^\circ$. At ω_f, $\Im[G(j\omega_f)] = 0$ (imaginary part is zero)

Step 2 - Find the gain of $G(j\omega)$ at $\omega = \omega_c$, i.e., $|G(j\omega_f)| = G$

Step 3 - Then gain margin in Decibels is $-20\log(G)$

State space model - back to temporal domain

State of a system: The set of variables that provides the future state and output of the system for a given input.



State variables: $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]$

Examples: Position, velocity, voltage, current, etc.

The space state representation is:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(\mathbf{t})$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(\mathbf{t})$$

Practice problems

Practice Exercises

Please refer to Lecture 15 for more examples pertaining to Lectures 1 to 14

Exercise 133

Calculate the magnitude and phase of

$$G(s) = \frac{1}{s + 10}$$

by hand for $\omega = 1,\ 2,\ 5,\ 10,\ 20,\ 50$ and 100 rad/s. Then, sketch the Bode plot of G(s)and compare the results. The Bode plot can be obtained in Matlab [1]

[1] Gain: 0.095, 0.0981, 0.0894, 0.0707, 0.0447, 0.0196, 0.0099
Phase: -5.71, -11.3, -26.6, -45, -63.4, -78.7, -84.3

Exercise 134

Sketch the Bode plot for the following open-loop transfer function and estimate the stability margins. After completing the and sketches, verify your results using Matlab.

$$L(s) = \frac{2000}{s(s+200)}$$

Matlab script

```
bode(tf([2000],[1 200 0]))
```

Exercise 135

Sketch the Bode plot for the following open-loop transfer function and estimate the stability margins. After completing the and sketches, verify your results using Matlab.

$$L(s) = \frac{s+2}{s(s+1)(s+5)(s+10)}$$

Matlab script:

```
s = tf([1 0],[1]);
bode((s+2)/(s*(s+1)*(s+5)*(s+10)))
```

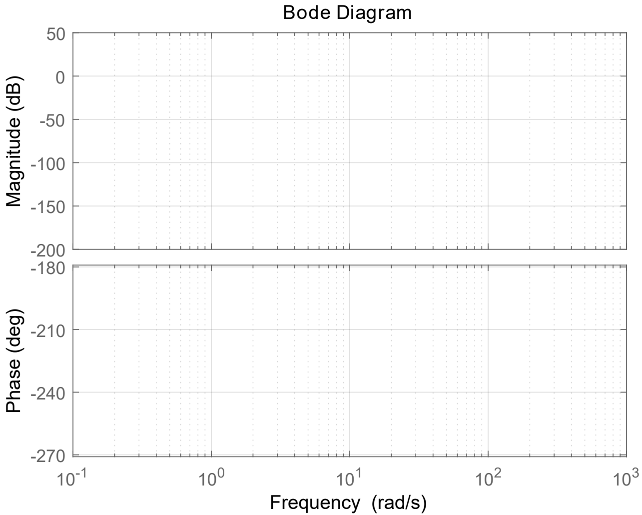
Exercise 136

Sketch the Bode plot for the following open-loop transfer function and estimate the stability margins. After completing the and sketches, verify your results using Matlab.

$$L(s) = \frac{1}{s^2(s+10)}$$

Matlab script:

```
s = tf([1 0],[1]);
bode(1/(s*s*(s+10)))
```



Bode Diagram
Magnitude (dB)
50
0
-50
-100
-150
-200
Phase (deg)
-180
-210
-240
-270
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)

Exercise 137

Sketch the Bode plot for the following open-loop transfer function and estimate the stability margins. After completing the and sketches, verify your results using Matlab.

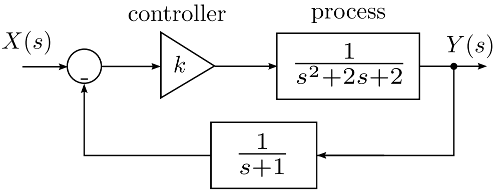
$$L(s) = \frac{s+2}{s(s+10)(s^2+2s+2)}$$

Matlab script:

```
s = tf([1 0],[1]);
bode((s+2)/(s*(s+10)*(s*s+2*s+2)))
```

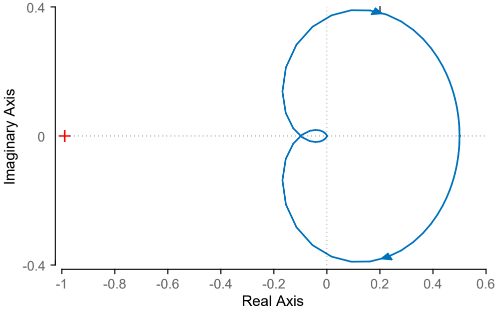
Exercise 138

Draw the Nyquist plot for the system shown. Using the Nyquist stability criterion, determine the range of *k* for which the system is stable.



Exercise 138 - continued

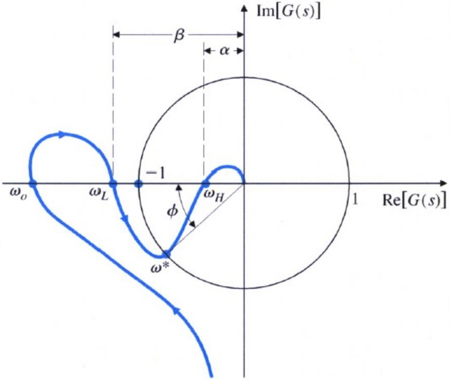
Answer



For positive k, note that the magnitude of the Nyquist plot as it crosses the negative real axis is 0.1, hence $k < 10$ for stability.

Exercise 139

The Nyquist plot for a control system resembles the one shown below. What is the phase margin(s)?[2]



[2] $-20\log(\alpha)$, $+20\log(\beta)$

Exercise 140

Determine the range of k for which the following system is stable by making a Bode plot for $k = 1$ and imagining the magnitude plot sliding up or down until instability results.

$$G(s) = \frac{k(s+3)}{s+30}$$

Verify your results using a very rough sketch of a root-locus plot.[3]

[3] Stable $\forall\ k > 0$

Exercise 141

Determine the range of k for which the following system is stable by making a Bode plot for $k = 1$ and imagining the magnitude plot sliding up or down until instability results.

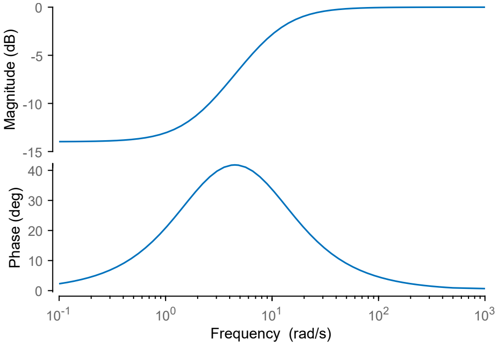
$$G(s) = \frac{k}{(s+10)(s+1)^2}$$

Verify your results using a very rough sketch of a root-locus plot.[4]

[4] Stable for $k < 242$

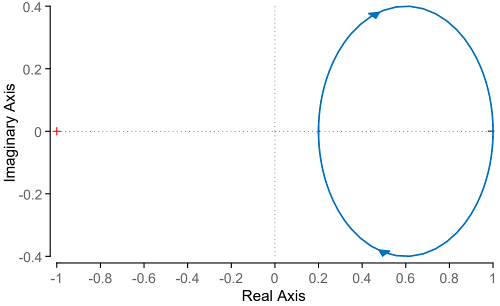
Exercise 142

The Bode plot of an unknown circuit has been obtained experimentally. Sketch the Nyquist plot of the system based on the Bode plot.



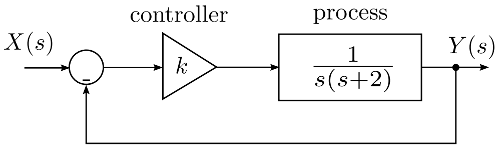
Exercise 142 - continued

Answer



Exercise 143

A feedback control system is shown. The closed-loop system is specified to have a phase margin of 40°. Determine k. [5]



[5] $k = 7.81$

Exercise 144

A two thank system is controlled by a motor adjusting the input valve and ultimately varying the output flow rate. The system has the transfer function

$$\frac{Q(s)}{I(s)} = P(s) = \frac{1}{s^3 + 10s^2 + 29s + 20}$$

Obtain a state variable model.

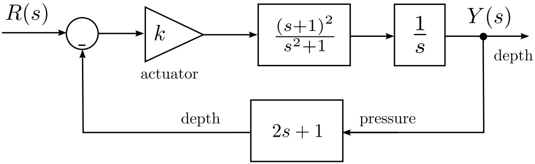
Exercise 144 - solution

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -20 & -29 & -10 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} p$$

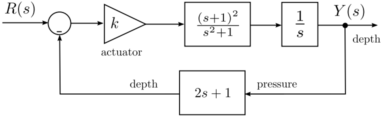
$$y = [\,1 \;\; 0 \;\; 0\,]x$$

Exercise 145

An automatic depth control system for a robot submarine is shown in the figure. The depth is measured by a pressure transducer. The gain of the stern place actuator is $k = 1$ when the vertical velocity is 25 m/s. Determine a state variable representation of the system.



Exercise 145 - continued

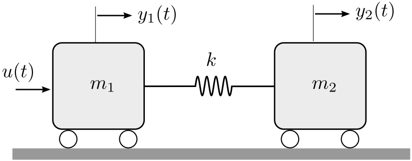


$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1/3 & -5/3 & -5/3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1/3 \end{bmatrix} r$$

$$y = [\,1 \;\; 2 \;\; 1\,]x$$

Exercise 146

A two mass system is shown. The rolling friction constant is b. Determine a state variable representation when the output variable.



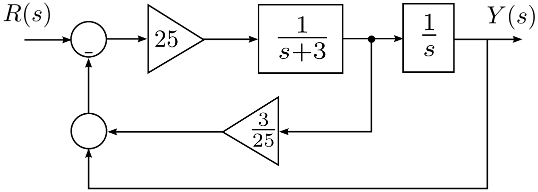
Exercise 146 - solution

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & -\frac{b}{m_1} & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & -\frac{b}{m_2} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [\,0 \;\; 0 \;\; 1 \;\; 0\,]x$$

Exercise 147

A system has block diagram shown. Determine a state variable model.



Exercise 147 - solution

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -25 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 25 \end{bmatrix} r$$

$$y = [\, 1 \;\; 0 \,] x$$

Exercise 148 - From 2018 final exam

Radio telescopes are used to study the radio frequency portion of the electromagnetic spectrum emitted by astronomical objects. One critical problem of driving large telescopes is the form of the system transfer function that has a structural resonance. A large telescope with a diameter of 20 meters, for example, is subject to large wind gust torques that can affect positioning accuracy. Consider an antenna, drive motor, and amplifier system with the following transfer function

$$G(s) = \frac{200}{(s+1)(s^2+5s+100)}$$

(a) Calculate the phase and magnitude of $G(s)$ at 1 rad/s.

(b) If the cross-over frequency is 1.8 rad/s, what is the phase margin?

(c) Draw the **approximate** Bode plot of $G(s)$. Based on the Bode plot, specify the approximate gain at the cut-off frequencies and at 10^3 rad/s (*8 marks*).

(d) Calculate the gain margin.

Exercise 149 - From 2018 deferred final exam

Radio telescopes are used to study the radio frequency portion of the electromagnetic spectrum emitted by astronomical objects. One critical problem of driving large telescopes is the form of the system transfer function that has a structural resonance. A large telescope with a diameter of 20 meters, for example, is subject to large wind gust torques that can affect positioning accuracy. Consider an antenna, drive motor, and amplifier system with the following transfer function

$$G(s) = 150\frac{s}{(s+0.7)^2(s^2+s+49)}$$

(a) Calculate the phase and magnitude of $G(s)$ at 1 rad/s.

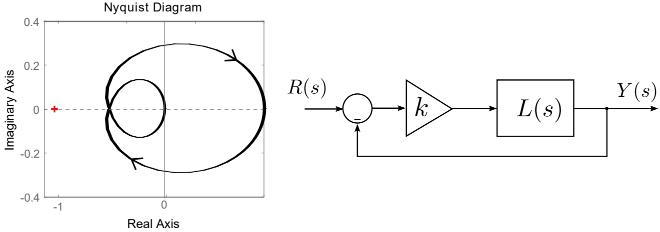
(b) If the cross-over frequency is 1.8 rad/s, what is the phase margin?

(c) Draw the **approximate** Bode plot of $G(s)$. Based on the Bode plot, specify the approximate gain at the cut-off frequencies and at 10^3 rad/s (*8 marks*).

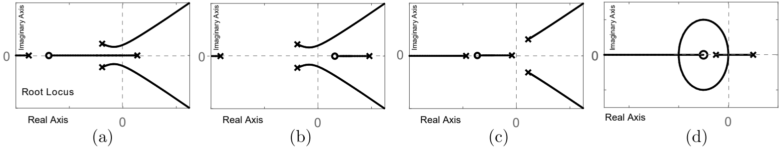
(d) Calculate the gain margin.

Exercise 150 - From 2018 final exam

The Nyquist plot below was obtained for the open-loop transfer function $kL(s)$ for a given positive value of k.



If $L(s)$ has **one unstable pole**, which of the following root-locus best approximates the root-locus of the closed-loop feedback system as $k \to \infty$?[6]



[6] Answer: (b), why?

Research opportunities

Students interested in part/full time research in mechatronics with applications to biomedical robotics:

bio MECHA TRONICS LAB

`www.biomechatronics.ca`

The end