MECE 3350U
Control Systems

Lecture 13
PID Controllers

Videos in this lecture

Lecture: `https://youtu.be/AVh-ryVbnxQ`

Exercise 66: `https://youtu.be/i_qbLtarxk4`

Exercise 67: `https://youtu.be/nLXte2lj9T8`

PID Demo 2: `https://youtu.be/MMo8TYb5v9Y`

Outline of Lecture 13

By the end of today's lecture you should be able to

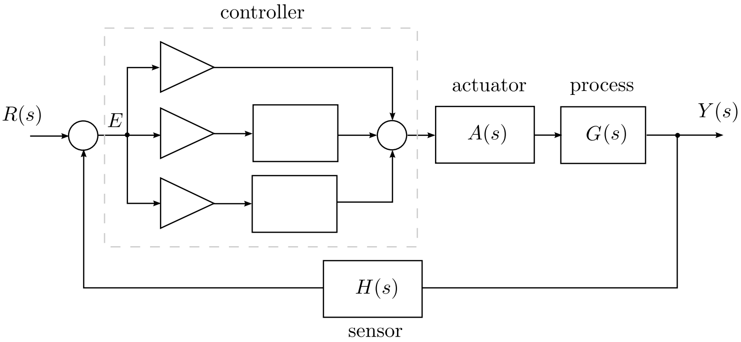
- Define a PID controller
- Understand the influence of the controller in the temporal response
- Understand the practical applications of PID control

Applications

Proportional-Integral-Derivative (PID) controllers are used in most automatic process control applications in industry.

More than 90% of industrial controllers are PID.

PID controller



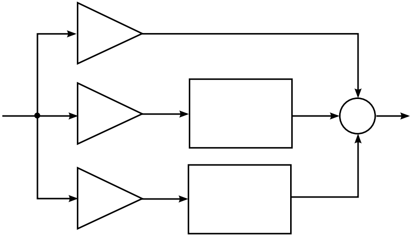
The command signal is a function of

→ The magnitude of the current error: proportional gain

→ The integral of the error over time: integral gain

→ The time rate change of the error: derivative gain

PID controller



$$u(t) = k_p e(t) + k_i \int_0^t e(t)dt + k_d \frac{d}{dt} e(t)$$

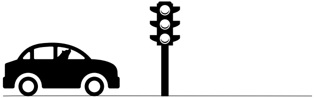
$$U(s) = k_p E(s) + k_i \frac{1}{s} E(s) + k_d s E(s)$$

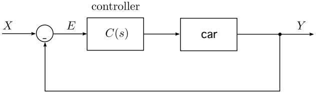
$$U(s) = \left(k_p + k_i \frac{1}{s} + k_d s\right) E(s)$$

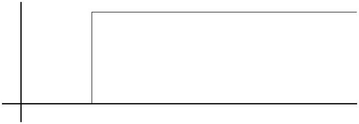
Proportional controller

$G(s) = k_p$, with $k > 0$.

$\dot{\theta}(t) = k_p e(t)$



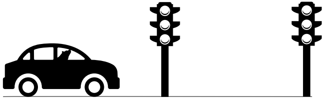


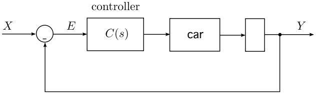


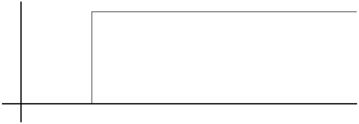
Proportional-derivative (PD) controller

$G(s) = k_p + k_d s$, with $k > 0$.

$\dot{\theta}(t) = k_p e(t) + k_d \frac{d}{dt} e(t)$







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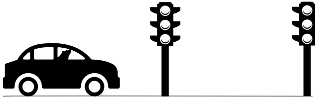
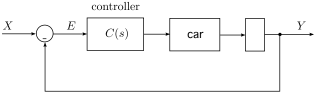
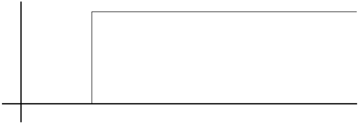
Proportional-derivative (PD) controller

$G(s) = k_p + k_d s$, with $k > 0$.

$\dot{\theta}(t) = k_p e(t) + k_d \frac{d}{dt} e(t)$



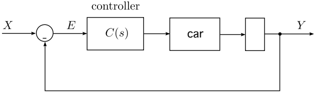




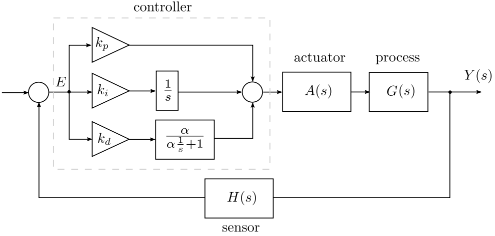
Proportional-integral-derivative (PID) controller

$$G(s) = k_p + k_d s + k_d \frac{1}{s}, \text{ with } k > 0.$$

$$\theta(t) = k_p e(t) + k_d \frac{d}{dt} e(t) + k_i \int e(t) dt$$



PID implementation



$$u(t) = k_p e(t) + k_i \int_0^t e(t)dt + k_d \frac{d}{dt} e(t)$$

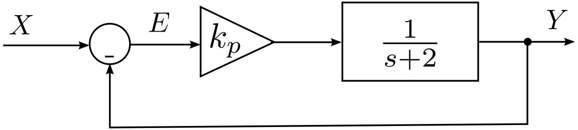
→ Proportional gain: Consider only the current error

→ Integral gain: Looks at the past error

→ Derivative gain: "Anticipates" the future error

PID controller

Effect of k_p



The transfer function is

$$T(s) = \frac{k_p}{s + k_p + 2} \quad (1)$$

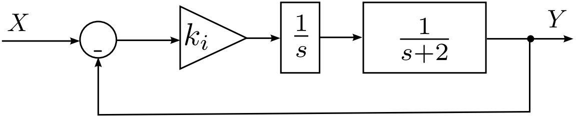
The time constant is $1/(k_p + 2)$.

The steady-state error for $x(t) = 1$ is

$$e(\infty) = u(t) - \lim_{s \to 0}[sF(s)] = 1 - \lim_{s \to 0} s \frac{k_p}{s + k_p + 2} \frac{1}{s} = 1 - \frac{k_p}{k_p + 2} \quad (2)$$

PID controller

Effect of the integrator and k_i on the transient response



The transfer function is

$$T(s) = \frac{k_i}{s(s+2)+k_i} = \frac{k_i}{s^2+2s+k_i} \tag{3}$$

where $\omega_n = \sqrt{k_i}$ and $\zeta = 1/\sqrt{k_i}$.

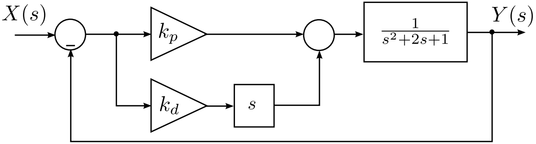
The **damping ratio decreases** with k_i.

Overshoot: $P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 100e^{-\pi/\sqrt{k_i-1}}$

The **overshoot increases** with k_i

PID controller

Effect of the derivative gain k_d



The transfer function is

$$T(s) = \frac{sk_d + k_p}{s^2 + (2 + k_d)s + 1 + k_p} \tag{4}$$

note the final value does not depend on k_d, thus it has **no effect** on the steady-state error.

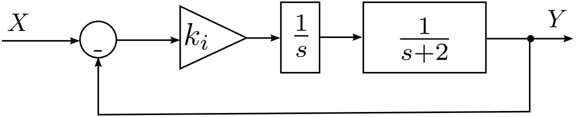
From the characteristic equation: $\zeta = (2 + k_d)/2(\sqrt{1 + k_p})$, $\omega_n = \sqrt{1 + k_p}$

The overshoot: $P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$ **decreases** with k_d.

The settling time: $Ts = 4/(\omega_n\zeta)$ **decreases** with k_d.

PID controller

Effect of the integrator on the steady-state error



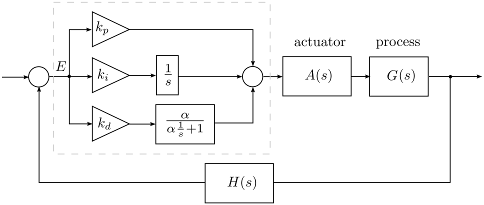
The transfer function is

$$T(s) = \frac{k_i}{s(s+2)+k_i} = \frac{k_i}{s^2+2s+k_i} \tag{5}$$

Steady-state error

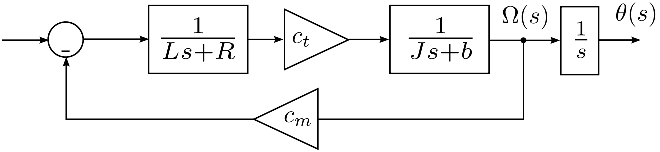
$$e(\infty) = u(t) - \lim_{s\to 0}[sF(s)] = 1 - \lim_{s\to 0} s\frac{k_i}{s(s+2)+k_i}\frac{1}{s} = 0 \tag{6}$$

PID gains



PID gain	Overshoot	Settling time	Steady-state error
Increasing k_p	Increases	Minimal impact	Decreases
Increasing k_i	Increases	Increases	Zero error
Increasing k_d	Decreases	Decreases	No impact

Speed control of a DC motor

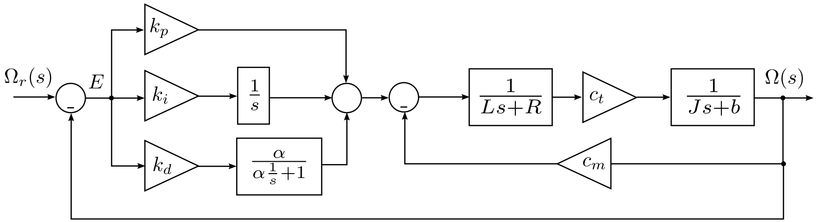


DC motor characteristics:

$J = 1.13 \times 10^{-2} Nm - sec^2/rad$, $b = 0.028 Nm - sec/rad$, $L = 0.1$ H

$R = 0.45$ Ohms, $c_t = 0.067$ Nm/A, $c_m = 0.067$ V-sec/rad

Speed control of a DC motor



$\alpha = 100$

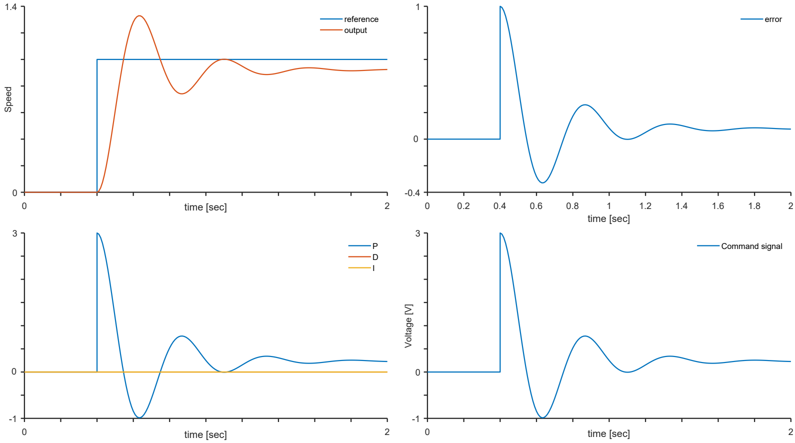
Proportional control: $k_p = 3,\ k_i = 0,\ k_d = 0$

Proportional-integral control: $k_p = 3,\ k_i = 15,\ k_d = 0$

PID control: $k_p = 3,\ k_i = 15,\ k_d = 0.3$

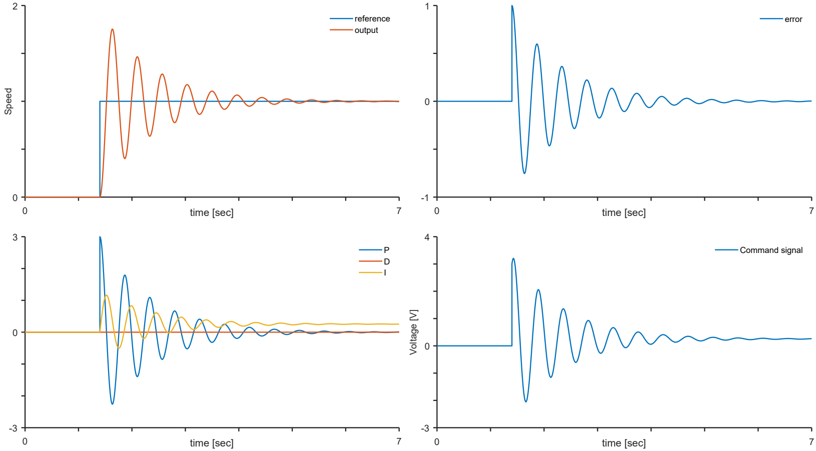
Speed control of a DC motor

Proportional control: $k_p = 3, \; k_i = 0, \; k_d = 0$



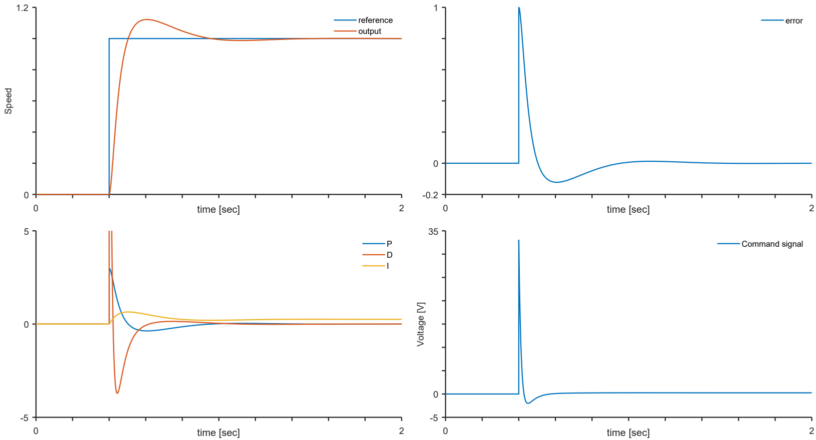
Speed control of a DC motor

Proportional-integral control: $k_p = 3,\ \ k_d = 15,\ \ k_d = 0$



Speed control of a DC motor

Proportional-integral-derivative control: $k_p = 3, \; k_i = 15, \; k_d = 0.3$



Exercise 66

A block diagram of a diesel engine driving load has an transfer function connecting the fuel valve setting $V(s)$ and the load shaft speed $Y(s)$ given by

$$\frac{Y(s)}{V(s)} = \frac{1}{0.01s^2 + 0.11s + 0.1}. \quad (7)$$

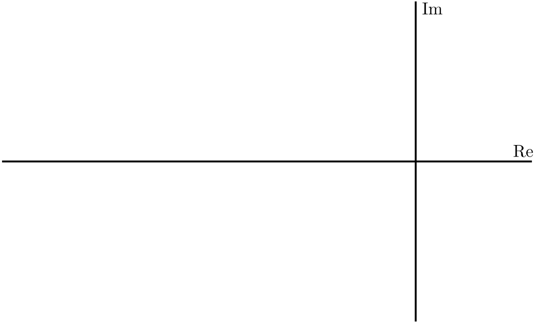
Speed control is provided by a PID controlled whose transfer function is

$$\frac{V(s)}{E(s)} = 5 + 0.3s + \frac{5}{sT} \quad (8)$$

By using the root-locus method, determine the minimum value τ if the complex closed loop dominant poles are to have a damping ratio not less than 0.7.

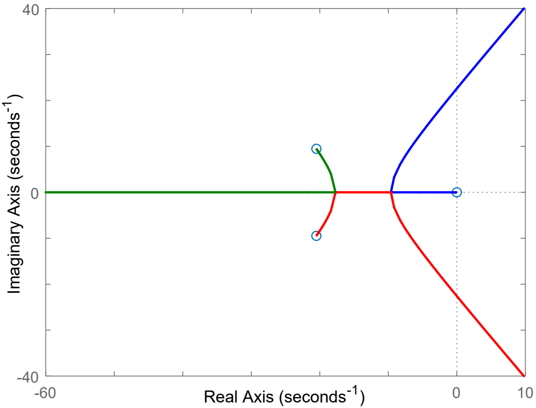
Exercise 66 - continued

$$1 + \tau \left(\frac{0.01s^3 + 0.41s^2 + 5.1s}{5} \right) = 0 \qquad (9)$$

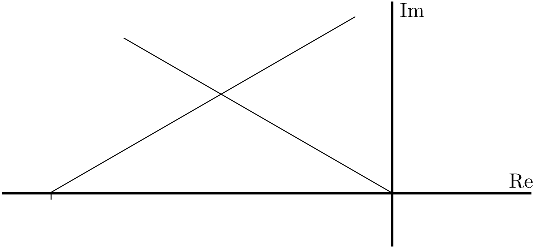


Exercise 66 - continued

$$1 + \tau \left(\frac{0.01s^3 + 0.41s^2 + 5.1s}{5} \right) = 0 \qquad (10)$$



Exercise 66 - continued



Exercise 66 - continued

$$s = -8.7 \pm 8.7j$$

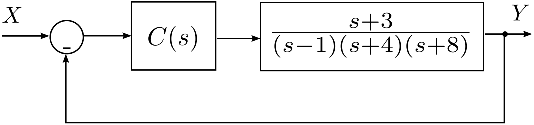
$$(0.01s^3 + 0.11s^2 + 0.1s)\tau + (0.3s^2 + 5s)\tau + 5 = 0$$

Centre of the asymptote is constant.

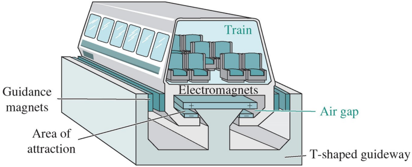
Thus we can estimate the location of the 3 pole as

Exercise 67

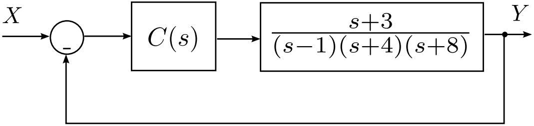
The air gap control system in a magnetically levitated high-speed train is controlled by a *PI* controller $G(s)$ have the same k_i and k_p gains.



Select the controller gain $k = k_i = k_p$ so that all of the complex poles have a damping ratio higher than 0.6.



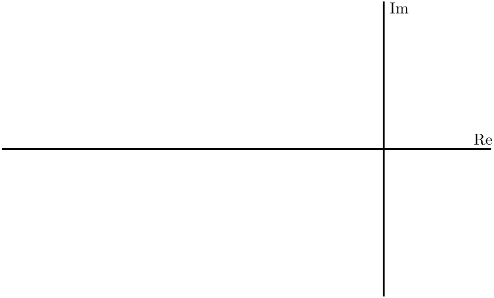
Exercise 67 - continued



$$C(s) =$$

Exercise 67 - continued

$$1 + k\frac{(s+1)(s+3)}{s(s-1)(s+4)(s+8)} = 0 \qquad (11)$$

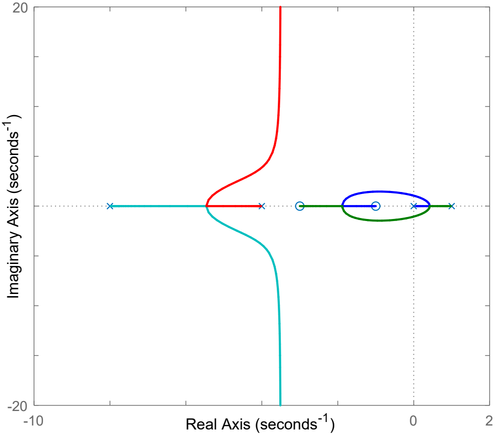


Asymptotes:

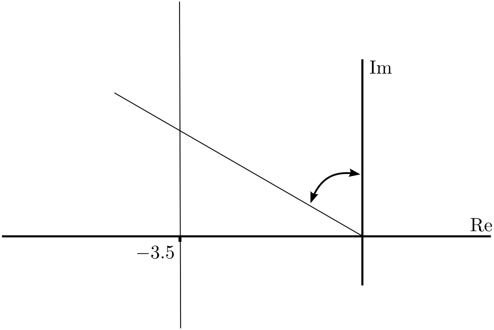
Centroid:

Exercise 67 - continued

$$1 + k\frac{(s+1)(s+3)}{s(s-1)(s+4)(s+8)} = 0 \quad (12)$$



Exercise 67 - continued



Skills check 36 - From a midterm examination

Circle the correct statement(s) regarding a PID controller.

(a) The greater the integral gain the lower the steady state error

(b) The higher the derivative gain the lower the steady state error

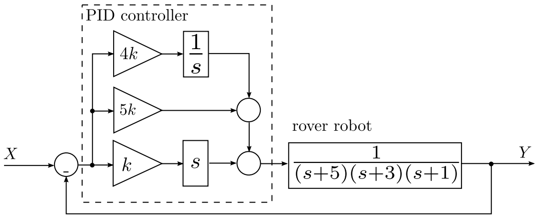
(c) The higher the integral gain the higher the overshoot

(d) The higher the derivative gain the lower the overshoot

(e) The greater the derivative gain, the faster a underdamped system reaches steady-state

Skills check 37 - From a midterm examination

With the exception of the Apollo program, most space exploration has been conducted with telerobotic space probes. The Russian Lunokhod-1 mission, for example, put a remotely driven rover on the moon, which was driven in real time by human operators on the ground. The model of such a system uses PID control the regulate the position of the teleoperated robot. The objective is to select a gain k that yields a reasonable overshoot.

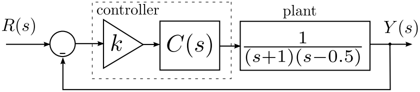


(a) Draw the **approximate** root-locus of the closed-loop feedback system as a function of k.

(b) Based on the root-locus, determine the value of k so that the dominant complex poles have a damping ratio of $\zeta = 0.707$. What is the settling time ?

Skills check 38 - From a final examination

A controller C(s) was developed for an unstable plant (shown below). The root locus of the feedback control system as a function of k is presented. Are the following statements true or false ?



(T) (F) $C(s)$ is a PID controller

(T) (F) $C(s)$ is a PD controller

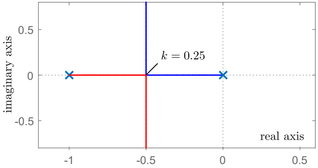
(T) (F) $C(s)$ is a PI controller

(T) (F) For $k < 0.25$, there is no overshoot

(T) (F) For $k \to \infty$, the system is unstable

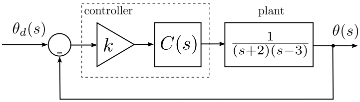
(T) (F) For $k > 0.25$, the system is underdamped

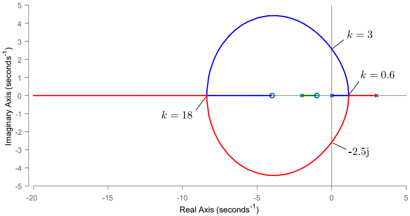
(T) (F) At the breakaway point the system is undamped



Skills check 39

A controller $C(s)$ was developed for the unstable plant shown and the respective root locus of the feedback control system as a function of k is presented.





Indicate whether the following statements about the closed-loop system are true or false. For each correct answer, +0.25 marks, each incorrect answer, -0.25 marks, D circled, 0 marks.

Skills check 39

T	F	D	$\rightarrow$ $C(s)$ is a PI controller
T	F	D	$\rightarrow$ $C(s)$ is a PID controller
T	F	D	$\rightarrow$ $C(s)$ is a PD controller
T	F	D	$\rightarrow$ $C(s)$ cannot stabilize the system
T	F	D	$\rightarrow$ The system is unstable for $k < 3$
T	F	D	$\rightarrow$ The system is stale for $k < 3$
T	F	D	$\rightarrow$ When $k = 18$ the system is critically damped
T	F	D	$\rightarrow$ When $k = 0.6$ the system is critically damped
T	F	D	$\rightarrow$ When $k = 3$ the system oscillates at 2.5 rad/s

Skills check 39

T	F	D	$\rightarrow$ When $k \rightarrow \infty$ the system is stable
T	F	D	$\rightarrow$ For $3 < k < 18$ the system is under-damped
T	F	D	$\rightarrow$ For $k > 18$ the time response is an exponential response
T	F	D	$\rightarrow$ A PI controller can stabilize the system
T	F	D	$\rightarrow$ A PI controller cannot stabilize the system
T	F	D	$\rightarrow$ A proportional controller can stabilize the system
T	F	D	$\rightarrow$ For $k > 18$ the system is overdamped
T	F	D	$\rightarrow$ For $k = 3$ the damping ratio is zero
T	F	D	$\rightarrow$ For $k \rightarrow 0$ the system is overdamped

Answers to skill check

S36 - (c), (d), and (e) are correct.

S37 - (a) use Matlab. (b) $k = 8.06$, $T_s = 2$ sec.

S38 - (F), (F), (T), (T), (F), (T), (F)

S39 - (F), (T), (F), (F), (T), (F), (T), (F), (T),
(T), (T), (T), (T), (F), (F), (T), (T), (F).

Next class...

- Tuning a PID controller