MECE 3350U
Control Systems

# Lecture 6

# Block Diagram Models

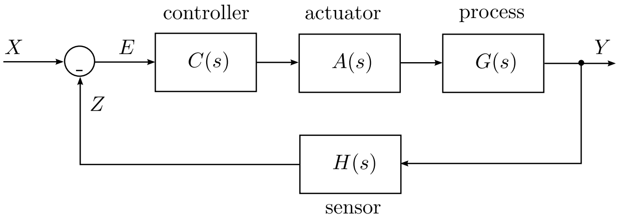
# Outline of Lecture 6

By the end of today's lecture you should be able to

- Represent a control system using block diagrams
- Simply block diagrams
- Find the open-loop transfer function of a closed-loop system

## Applications

What the transfer function of the closed-loop system shown ?



## Applications

The position control system for a spacecraft platform is governed by the following equations:

$$\frac{d^2p(t)}{dt^2} + 2\frac{dp(t)}{dt} + 4p(t) = \theta(t)$$
$$v_1(t) = r(t) - p(t)$$
$$\frac{d\theta(t)}{dt} = 0.5v_2(t)$$
$$v_2(t) = 8v_1(t)$$

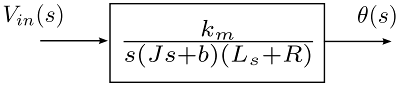
$r(t)$: desired position
$p(t)$: current position
$v_1(t)$: amplifier input voltage
$v_2(t)$: amplifier output voltage
$\theta(t)$: motor shaft position

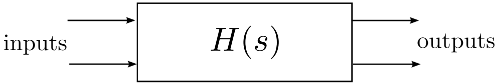
How can we represent the system using a block diagram ?

# Block diagrams

→ Represent the relationship of a system variables graphically.

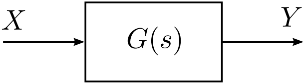
→ Example: The relation between the input voltage and and the position of a DC motor



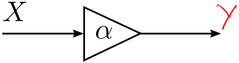
## Basic building elements

Transfer function



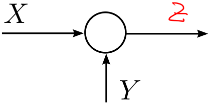

$Y(s) = X(s)\,G(s)$

Gain

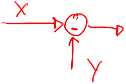



$Y(s) = \alpha X(s)$

Sum




$Z(s) = X + Y$



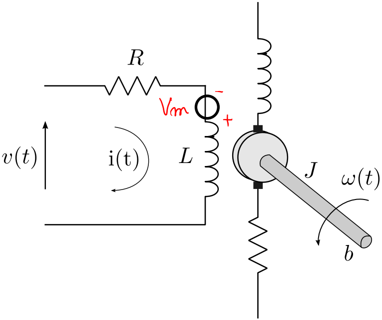

$Z = X - Y$

## Block diagram of a DC motor

→ Electric circuit characteristics

$V(s) = (R + Ls)\, I(s) + V_m(s)$ (1)

Back EMF




→ Back electromagnetic force voltage

$V_m(s) = K_m\, \Omega(s)$ (2)

speed

$K_m \to$ constant

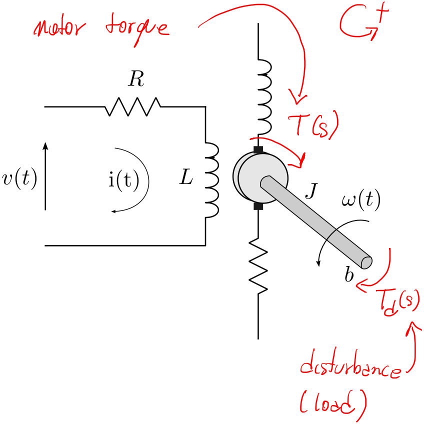
$$V(s) = (R + Ls)I(s) + \omega(s)k_m \to I(s) = \frac{V(s) - V_m(s)}{R + Ls}$$

## Block diagram of a DC motor

→ Mechanical characteristics

$$T(s) - T_d(s) - b\,\Omega(s) = Js\,\Omega(s)$$

$$\boxed{\Omega(s) = \frac{T(s) - T_d(s)}{Js + b}} \qquad (3)$$




→ Torque constant

$$T(s) = K_i\, I(s) \qquad (4)$$

↳ torque constant

$$T(s) = (Js^2 + bs)\theta(s) + T_d \rightarrow \theta(s) = \frac{I(s)k_i - T_d}{Js^2 + bs} \rightarrow \omega(s) = \frac{I(s)k_i - T_d}{Js + b}$$

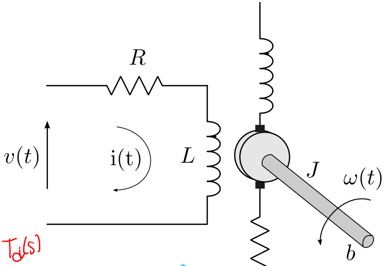
# Block diagram of a DC motor

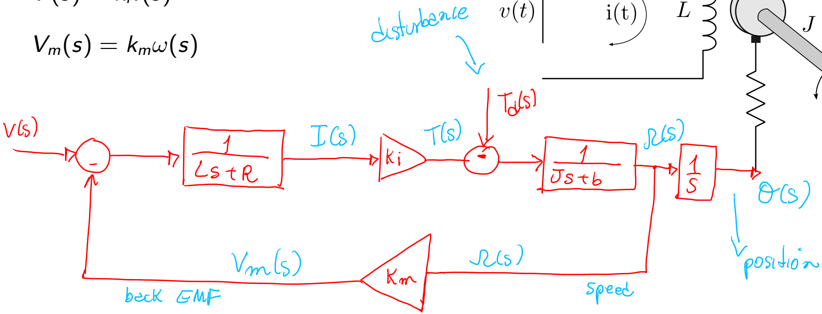
$$I(s) = \frac{V(s) - V_m(s)}{Ls + R}$$

$$\omega(s) = \frac{T(s) - T_d(s)}{Js + b}$$

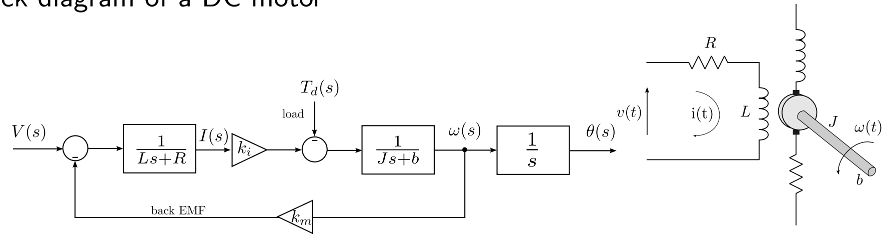
$$T(s) = k_i I(s)$$

$$V_m(s) = k_m \omega(s)$$



# Block diagram of a DC motor

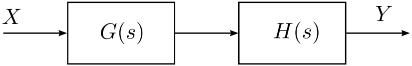



Simulation with Matlab - Simulink

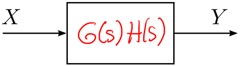
Evaluate the step response of the motor

# Basic operations

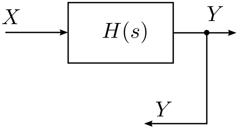
Combining blocks in cascade

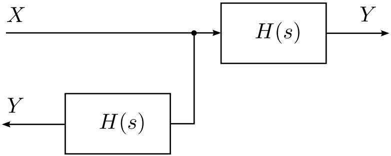



$Y = G(s)H(s)X(s)$




Moving a pickoff point ahead of a block

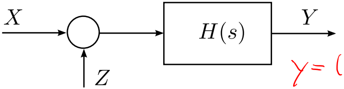


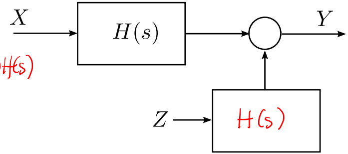
## Basic operations

Moving a summing point ~~behind~~ ahead a block

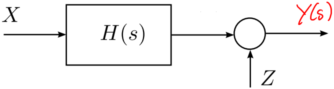
$Y = X\,H(s) + Z\,H(s)$



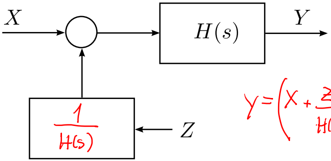

$Y = (X+Z)H(s)$




Moving a summing point ~~ahead~~ behind of a block

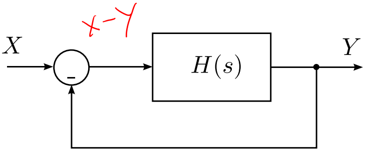


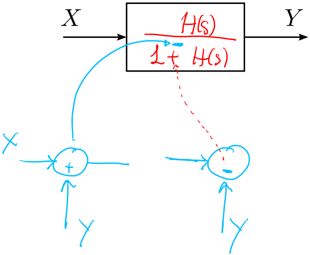

$Y = X\,H(s) + Z$




$Y = \left(X + \frac{Z}{H(s)}\right) H(s)$

## Eliminating a feedback loop



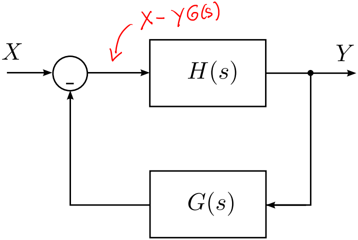

$$Y = (X - Y)\,H(s)$$

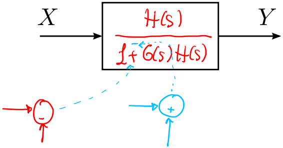
$$Y + Y\,H(s) = X\,H(s)$$

$$Y\left[1 + H(s)\right] = X\,H(s)$$

$$Y = \frac{H(s)}{1 + H(s)}$$

## Eliminating a feedback loop




$$Y = [X - Y\,G(s)]\,H(s)$$

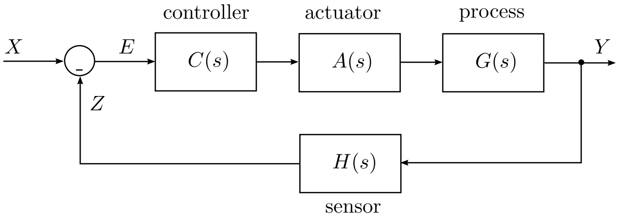
$$Y + Y\,G(s)H(s) = X\,H(s)$$

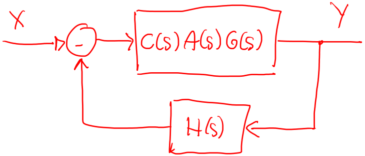
$$Y[1 + G(s)H(s)] = X\,H(s)$$

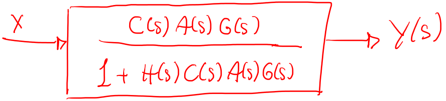
$$\boxed{Y = \frac{H(s)}{1 + G(s)H(s)}\,X(s)}$$

## Example 1

Find the open-loop transfer function of the closed-loop system shown.

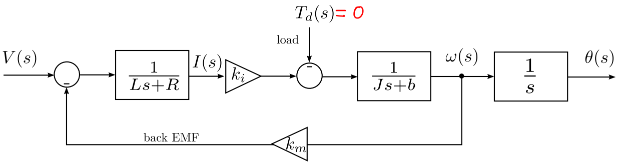


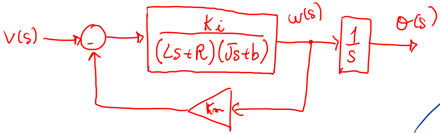




$$\frac{Y(s)}{X(s)} = \frac{C(s)A(s)G(s)}{1 + H(s)C(s)A(s)G(s)} \quad //$$

# Example 2 - DC motor

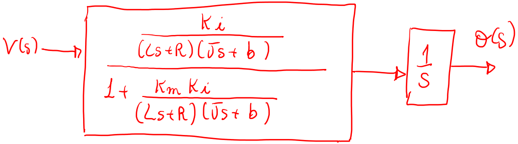



If $T = 0$, what is the transfer function $\theta(s)/V(s)$ ?

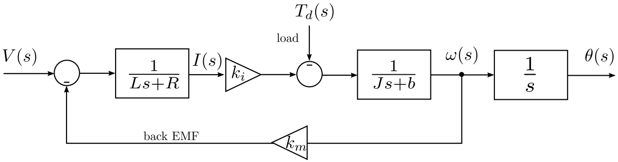



$$\frac{V(s)}{\theta(s)} = \left(\frac{K_i}{(Ls+R)(Js+b)+K_iK_m}\right)\frac{1}{s}$$

//



## Example 2 - DC motor




$$G(s) = \frac{\theta(s)}{V(s)} = \frac{k_i}{s[(Ls+R)(Js+b)+k_i k_m]} \tag{1}$$

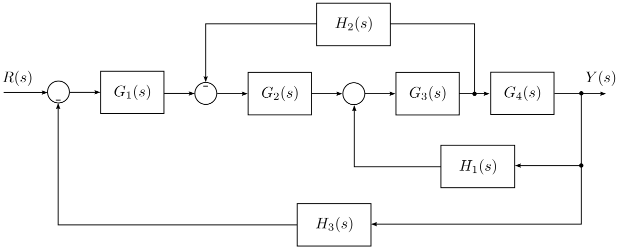
Sometimes the armature time constant $\tau_a = L/R$ is negligible, thus

$$G(s) \approx \frac{\theta(s)}{V(s)} = \frac{k_i}{s[R(Js+b)+k_i k_m]} = \frac{k_i/(Rb+K_i K_m)}{s(\tau s+1)} \tag{2}$$

where $\tau = \frac{RJ}{Rb+K_i K_m}$

# Exercise 23

Find the transfer function $Y(s)/R(s)$ of the system shown.

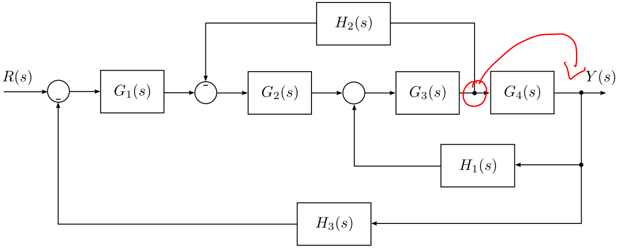


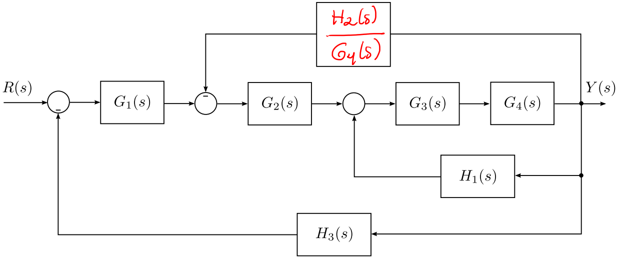

**Procedure**:

→ Simply the block diagram

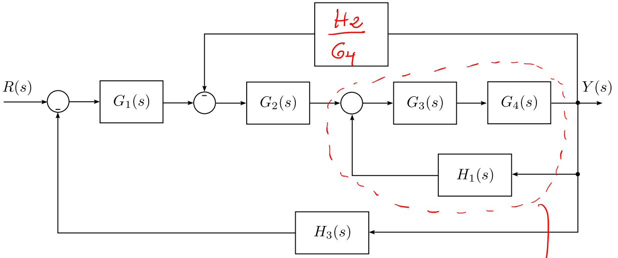
→ Calculate the closed-loop transfer function

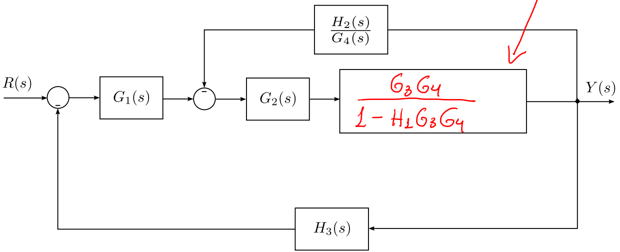
# Exercise 23 - continued



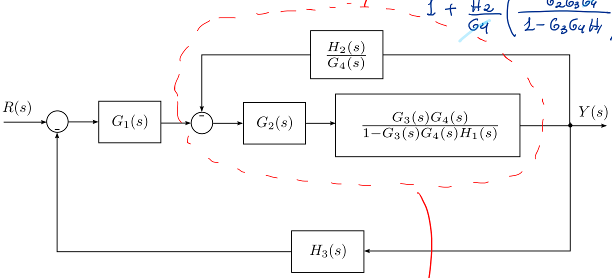
# Exercise 23 - continued

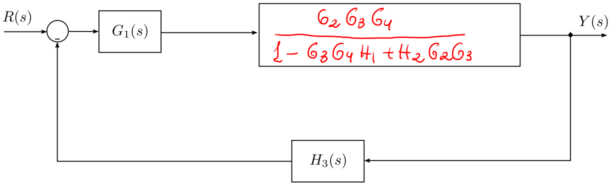


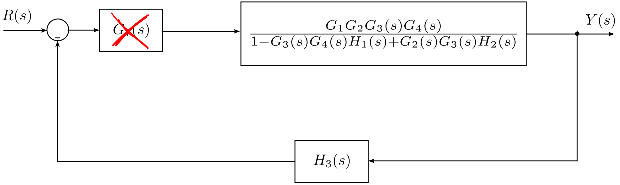
# Exercise 23 - continued

$$\frac{\frac{G_2G_3G_4}{1-G_3G_4H_1}}{1+\frac{H_2}{\cancel{G_4}}\left(\frac{G_2G_3\cancel{G_4}}{1-G_3G_4H_1}\right)}$$



# Exercise 23 - continued

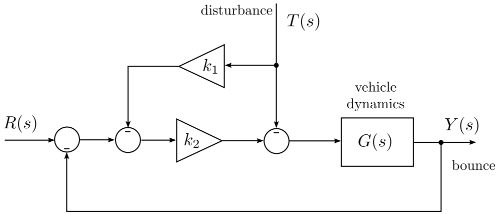



$$\frac{\dfrac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2}}{1 + H_3\left(\dfrac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2}\right)}$$

$$\Rightarrow Y(s) = \frac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2+H_3G_1G_2G_3G_4}\, X(s)$$

## Exercise 24

An active suspension system can be controlled by a sensor that looks ahead at the road conditions. An example that can accommodate road bumps is shown in the figure. Find the gain $k_1$ so that the vehicle does not bounce when the desired deflection is $R(s) = 0$ and the disturbance is $T(s)$.




**Procedure**:

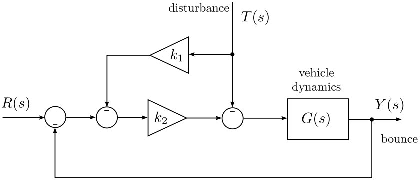
$\rightarrow$ Find the transfer function from $T(s)$ to $R(s)$

$\rightarrow$ Set the bounce to zero ($Y(s) = 0$)

$\rightarrow$ Calculate $k_1$

## Exercise 24 - continued

Set $R(s)=0$, (no bumps)




$$Y(s) = \left[-T(s) - T(s)K_1K_2 - Y(s)K_2\right]G(s)$$

$$Y(s) + Y(s)G(s)K_2 = -T(s)G(s)\left[1+K_1K_2\right]$$

$$Y(s) = \frac{-T(s)G(s)\left[1+K_1K_2\right]}{1+G(s)K_2}$$

No bumps $\longrightarrow$ $Y(s)=0$

thus $1+K_1K_2=0$

$$\boxed{K_1 = -\frac{1}{K_2}}$$

# Exercise 25

The position control system for a spacecraft platform is governed by the following equations:

$$\frac{d^2p(t)}{dt^2} + 2\frac{dp(t)}{dt} + 4p(t) = \theta(t)$$
$$v_1(t) = r(t) - p(t)$$
$$\frac{d\theta(t)}{dt} = 0.5v_2(t)$$
$$v_2(t) = 8v_1(t)$$

$r(t)$: desired position
$p(t)$: current position
$v_1(t)$: amplifier input voltage
$v_2(t)$: amplifier output voltage
$\theta(t)$: motor shaft position

**To do**:

→ Sketch a block diagram of the system

→ Find the transfer function $P(s)/R(s)$

# Exercise 25 - continued

① $$\frac{d^2p(t)}{dt^2} + 2\frac{dp(t)}{dt} + 4p(t) = \theta(t)$$

② $$v_1(t) = r(t) - p(t)$$

$$\frac{d\theta(t)}{dt} = 0.5v_2(t)$$

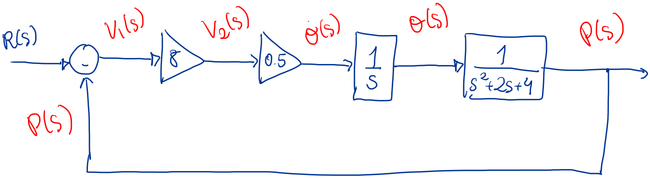
$$v_2(t) = 8v_1(t)$$

① $(s^2 + 2s + 4)\,P(t) = \theta(s)$

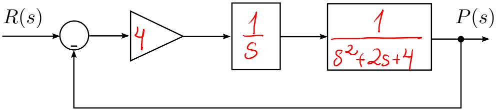
② $V_1(s) = R(s) - P(t)$

③ $\theta(s)\,s = 0.5\,V_2(s)$

④ $V_2(s) = 8\,V_1(s)$



## Exercise 25 - continued



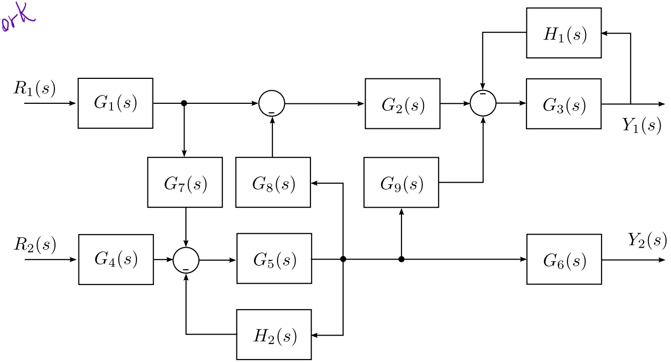

$$\frac{P(s)}{R(s)} = \frac{\frac{4}{s}\,\frac{1}{s^2+2s+4}}{1+\frac{4}{s}\,\frac{1}{s^2+2s+4}}$$

$$\boxed{\frac{P(s)}{R(s)} = \frac{4}{s^3+2s^2+4s+4}}$$

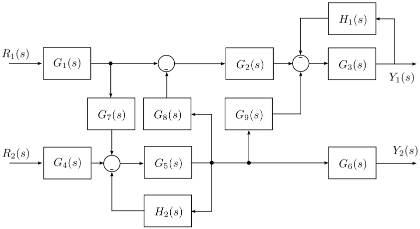
## Exercise 26

Compute the transfer function $Y_1(s)/R_2(s)$. Hint: Using the principle of superposition, set $R_1(s) = 0$.

Homework



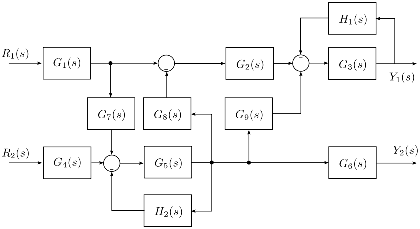
# Exercise 26 - continued




Solution:

$$\frac{Y_1}{R_2} = \frac{G_2 G_3 G_4 G_5 G_8 + G_3 G_4 G_5 G_9}{1 + G_3 H_1 + G_5 H_2 + G_3 G_5 H_1 H_2} \quad //$$

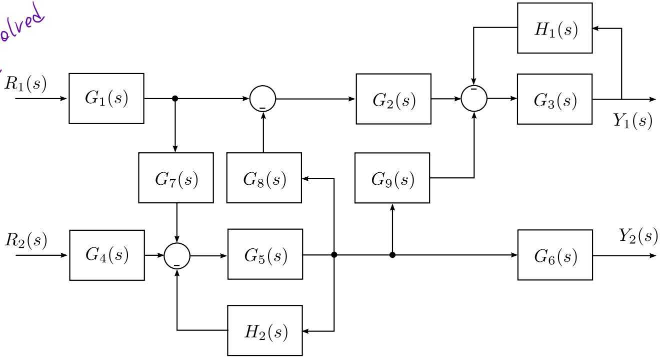
# Exercise 26 - continued



# Exercise 27

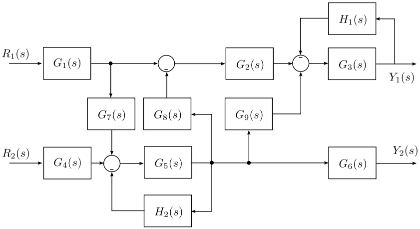
Compute the transfer function $Y_2(s)/R_1(s)$. Hint: Using the principle of superposition, set $R_2(s) = 0$.

Homework
May be solved during a tutorial.

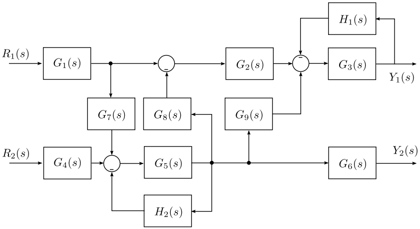


# Exercise 27 - continued

tutorial

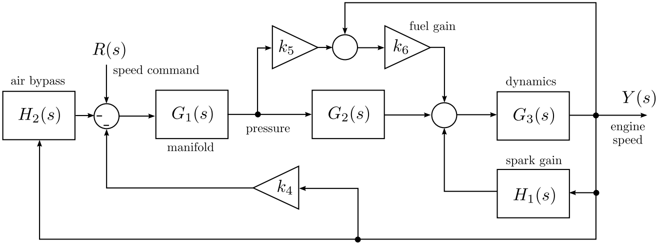


# Exercise 27 - continued

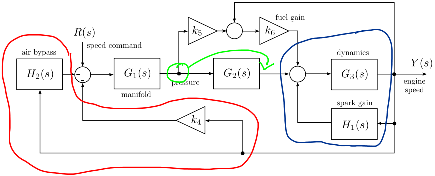


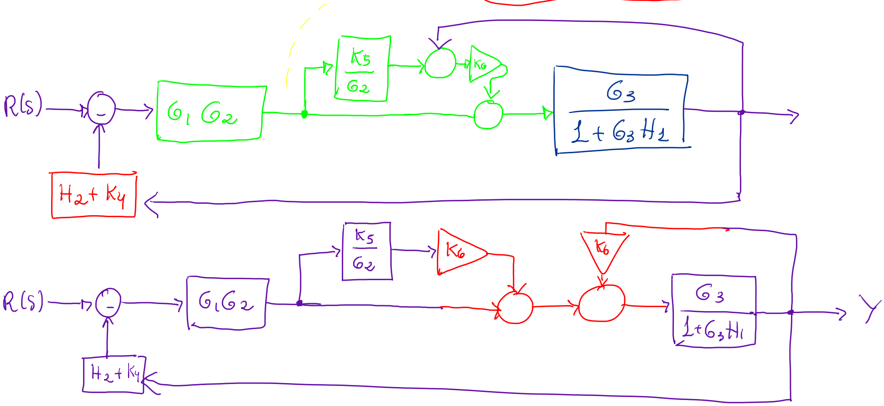
# Exercise 28

Compute the transfer function $Y(s)/R(s)$ for the idle-speed control system for a fuel-injected engine as shown in the block diagram.

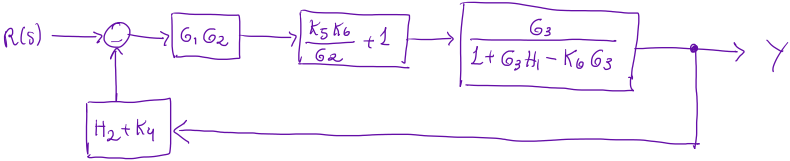


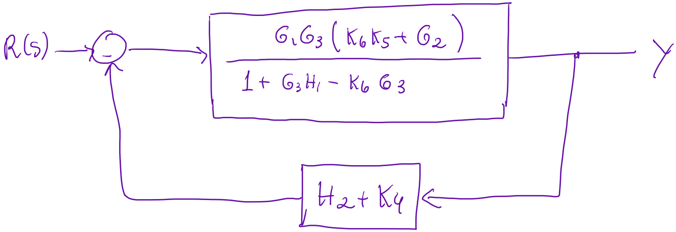
# Exercise 28 - continued



# Exercise 28 - continued



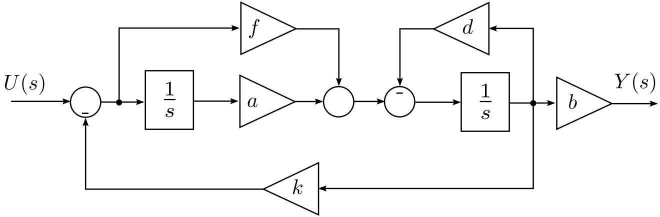



## Exercise 28 - continued

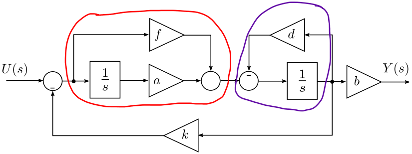
$$\frac{Y(s)}{R(s)} = \frac{G_1 G_3 [K_5 K_6 + G_2)}{1 + G_3 (H_1 - K_6) + G_1 G_3 (K_5 K_6 + G_2)(K_4 + H_2)} \quad //$$

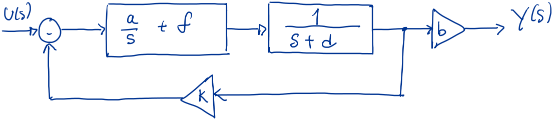
# Exercise 29

Compute the transfer function $Y(s)/U(s)$ for block diagram shown.



# Exercise 29 - continued




$$\frac{Y(s)}{U(s)} = \frac{\left(\frac{a}{s}+f\right)\left(\frac{1}{s+d}\right)}{1 + K\left(\frac{a}{s}+f\right)\left(\frac{1}{s+d}\right)}\, b \quad = b\,\frac{a+fs}{s^2+ds+K(a+fs)} \quad //$$

## Next class...

- Steady state error