MECE 3350U
Control Systems

Lecture 22

Final Examination Review and Practice Exercises

Final examination

- When: Dec 14, 15:30-18:30
- Where: Gym.
- Seating assignments: **Section 15**: A, C, D, F, **Section 21**: G, I, J, L
- What: Lectures 2 to 22 (evenly distributed)
- Prepare your formula sheet (1 page, letter size, both sides)
- Bring a photo ID or student card.
- Exam problems are in line with those solved in class, tutorials, and assignments.

Formula sheet

Prepare your own formula sheet

Everything must be handwritten

Your formula sheet cannot exceed 1 page (letter size), both sides.

Please write your name/student ID on the formula sheet

Gain and phase - review

For a generic transfer function $G(s)$

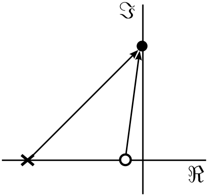
$$G(s) = k\frac{\prod_{i=1}^{n}(s+z_i)}{\prod_{k=1}^{m}(s+p_k)}$$

we can evaluate the **phase** at a frequency ω by letting $s = j\omega$.

The phase is

$$\angle G(j\omega) = \angle|k| + \sum_{i=1}^{n}\angle(j\omega + z_i) - \sum_{k=1}^{m}\angle(j\omega + p_k)$$

where $\angle(j\omega + a) = \tan^{-1}\omega/a$



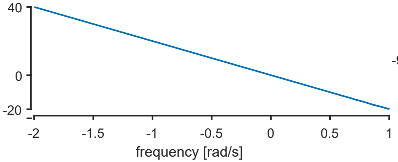
Bode plot building blocks

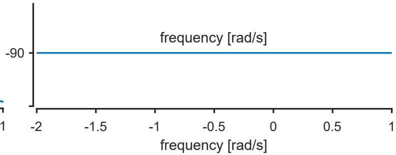
1 - Constant gain

→ Gain: $|k|$ or $20\log(|k|)$

→ Phase: $\phi = 0 \ \forall \ \omega$ if $k > 0$, -180° otherwise

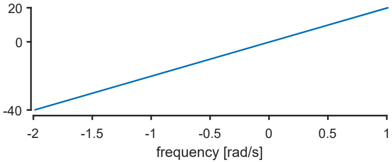
2 - Pole at the origin



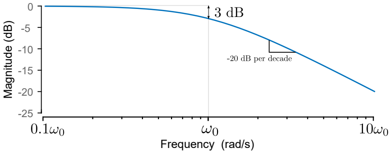


Bode plot building blocks

3 - Zero at the origin

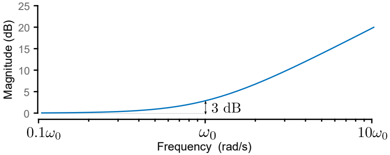


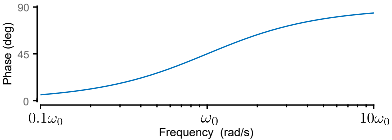
4 - Real pole: $G(s) = \frac{1}{\frac{s}{\omega_0}+1}$, $\omega_0 \in \Re^*$



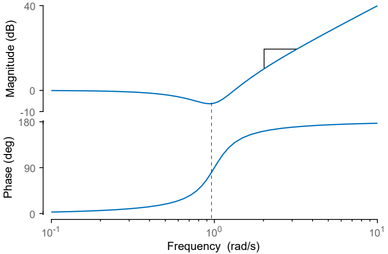
Bode plot building blocks

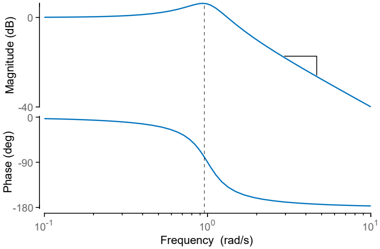
5 - Real zero: $G(s) = \frac{s}{\omega_0} + 1,\ \omega_0 \in \Re^*$



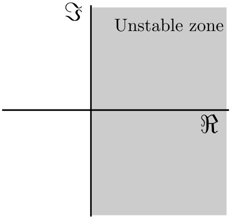


6 - Imaginary zeros or poles:





Open loop *vs* closed loop stability



Open-loop stability

$$T(s) = C(s)G(s)$$

$\rightarrow$ Evaluate the location of the **poles** of $C(s)G(s)$

Closed-loop stability

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}$$

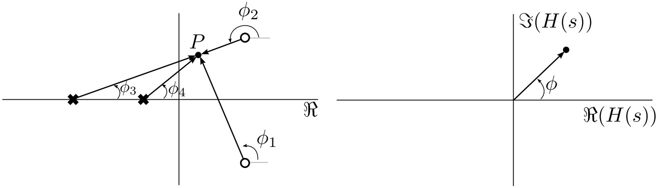
$\rightarrow$ Evaluate the location of the **zeros** of $1 + C(s)G(s)$

Example: If $C(s)G(s) = \frac{s+a}{s+b}$

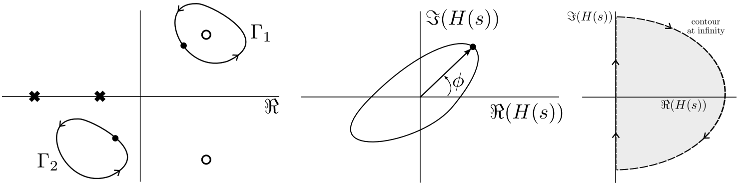
$\rightarrow$ Open-loop stable if $C(s)G(s)$ has real negative **poles**: i.e., $b > 0$

$\rightarrow$ Closed-loop stable if $1 + C(s)G(s)$ has real negative **zeros**:

Cauchy's argument principle



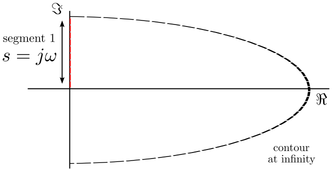
A contour map of a complex function will encircle the origin $N = Z - P$ times, where Z is the number of zeros and P is the number of poles of the function inside the contour.



The Nyquist Stability Criterion

A open-loop transfer function $L(s)$ is closed-loop stable if and only if the number of counterclockwise encirclements of the $-1+0j$ point is equal to the number of poles of $L(s)$ with positive real parts

$$Z = N + P$$



Nyquist plot

→ The contour at infinity maps to a single point

→ $\omega = 0$ (starting point)

→ $\omega \to \infty$

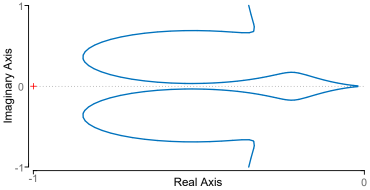
→ Point where the plot crosses the real and imaginary axis

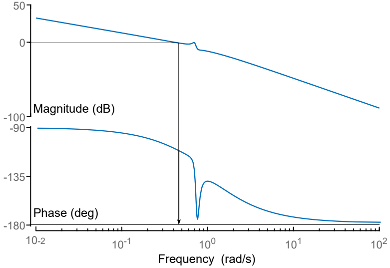
Gain and phase margins

The characteristic equation of a closed loop system with unit feedback is

$$1 + C(s)G(s) = 0$$

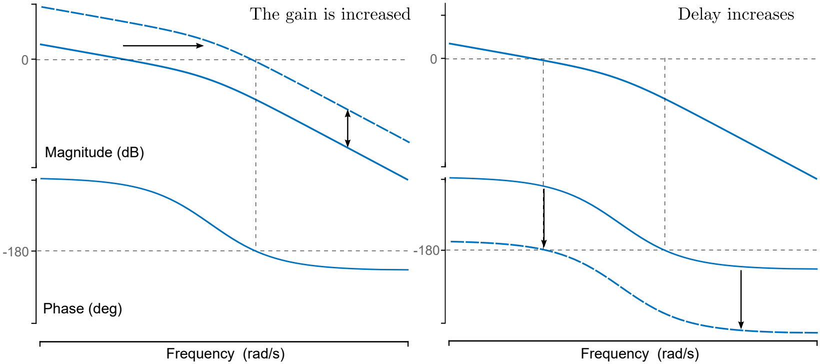
If $|C(s)G(s)| = 1$ and $\angle|C(s)G(s)| = \pm 180^\circ$, the characteristic equations is zero





Stability margin: How far the system if from $-1 + 0j$ or $1\angle 180^\circ$

Gain and phase margins



Phase and gain margin

Phase margin

Step 1 - Find the crossover frequency (0 dB). At the crossover frequency $\omega = \omega_c$, the magnitude is 1

Step 2 - Find the phase of $G(j\omega)$ at ω_c for ω_c found in Step 1, i.e. $\angle G(j\omega_c)$

Step 3 - The margin phase is $180 - |\phi|$

Gain margin

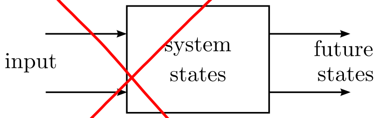
Step 1 - Find the frequency ω_f where $\angle|G(j\omega)| = -180^\circ$. At ω_f, $\Im[G(j\omega_f)] = 0$ (imaginary part is zero)

Step 2 - Find the gain of $G(j\omega)$ at $\omega = \omega_c$, i.e., $|G(j\omega_f)| = G$

Step 3 - Then gain margin in Decibels is $-20\log(G)$

State space model - back to temporal domain

State of a system: The set of variables that provides the future state and output of the system for a given input.



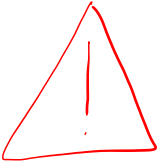
State variables: $\mathbf{x}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]$

Examples: Position, velocity, voltage, current, etc.

The space state representation is:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(\mathbf{t})$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(\mathbf{t})$$

Practice problems



Practice Exercises

Please refer to Lecture 15 for more examples pertaining to Lectures 1 to 14

Exercise 128

Calculate the magnitude and phase of

$$G(s) = \frac{1}{s + 10}$$

by hand for $\omega = 1,\ 2,\ 5,\ 10,\ 20,\ 50$ and 100 rad/s. Then, sketch the Bode plot of G(s)and compare the results. The Bode plot can be obtained in Matlab [1]

[1] Gain: 0.095, 0.0981, 0.0894, 0.0707, 0.0447, 0.0196, 0.0099
Phase: -5.71, -11.3, -26.6, -45, -63.4, -78.7, -84.3

Exercise 129

Sketch the Bode plot for the following open-loop transfer function and estimate the stability margins. After completing the and sketches, verify your results using Matlab.

$$L(s) = \frac{2000}{s(s+200)}$$

Matlab script

```
bode(tf([2000],[1 200 0]))
```

Exercise 130

Sketch the Bode plot for the following open-loop transfer function and estimate the stability margins. After completing the and sketches, verify your results using Matlab.

$$L(s) = \frac{s+2}{s(s+1)(s+5)(s+10)}$$

Matlab script:

```
s = tf([1 0],[1]);
bode((s+2)/(s*(s+1)*(s+5)*(s+10)))
```

Exercise 131

Sketch the Bode plot for the following open-loop transfer function and estimate the stability margins. After completing the and sketches, verify your results using Matlab.

$$L(s) = \frac{1}{s^2(s+10)}$$

Matlab script:

```
s = tf([1 0],[1]);
bode(1/(s*s*(s+10)))
```

Exercise 132

Sketch the Bode plot for the following open-loop transfer function and estimate the stability margins. After completing the and sketches, verify your results using Matlab.

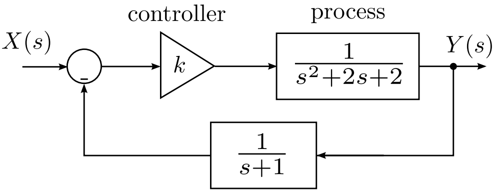
$$L(s) = \frac{s+2}{s(s+10)(s^2+2s+2)}$$

Matlab script:

```
s = tf([1 0],[1]);
bode((s+2)/(s*(s+10)*(s*s+2*s+2)))
```

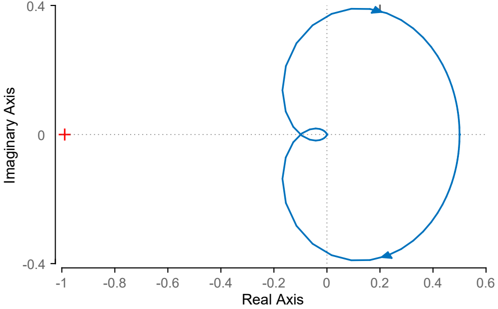
Exercise 133

Draw the Nyquist plot for the system shown. Using the Nyquist stability criterion, determine the range of k for which the system is stable.



Exercise 133 - continued

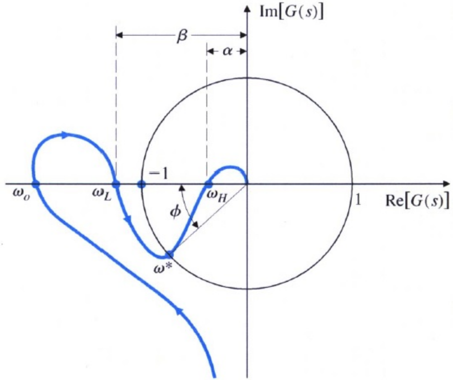
Answer

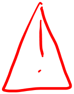


For positive k, note that the magnitude of the Nyquist plot as it crosses the negative real axis is 0.1, hence $k < 10$ for stability.

Exercise 134

The Nyquist plot for a control system resembles the one shown below. What is the phase margin(s)?[2]





[2] $-20\log(\alpha)$, $+20\log(\beta)$

Exercise 135

Determine the range of k for which the following system is stable by making a Bode plot for $k = 1$ and imagining the magnitude plot sliding up or down until instability results.

$$G(s) = \frac{k(s+3)}{s+30}$$

Verify your results using a very rough sketch of a root-locus plot.[3]

[3] Stable $\forall\ k > 0$

Exercise 136

Determine the range of k for which the following system is stable by making a Bode plot for $k = 1$ and imagining the magnitude plot sliding up or down until instability results.

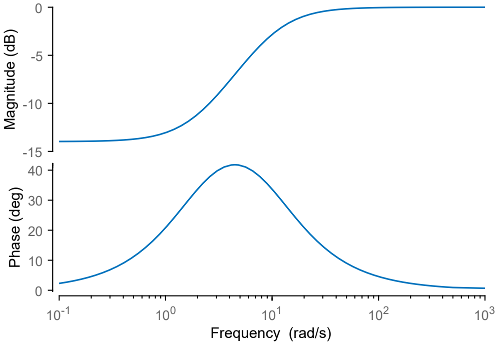
$$G(s) = \frac{k}{(s+10)(s+1)^2}$$

Verify your results using a very rough sketch of a root-locus plot.[4]

[4] Stable for $k < 242$

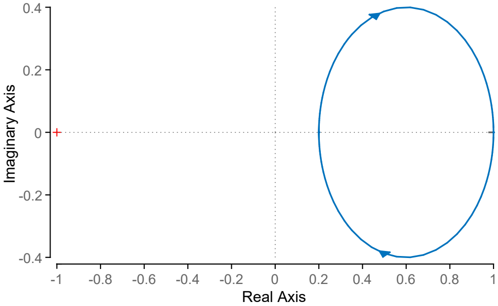
Exercise 137

The Bode plot of an unknown circuit has been obtained experimentally. Sketch the Nyquist plot of the system based on the Bode plot.



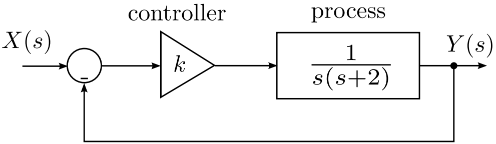
Exercise 137 - continued

Answer



Exercise 138

A feedback control system is shown. The closed-loop system is specified to have a phase margin of 40°. Determine k. [5]



[5] $k = 7.81$

Exercise 139

A two thank system is controlled by a motor adjusting the input valve and ultimately varying the output flow rate. The system has the transfer function

$$\frac{Q(s)}{I(s)} = P(s) = \frac{1}{s^3 + 10s^2 + 29s + 20}$$

Obtain a state variable model.

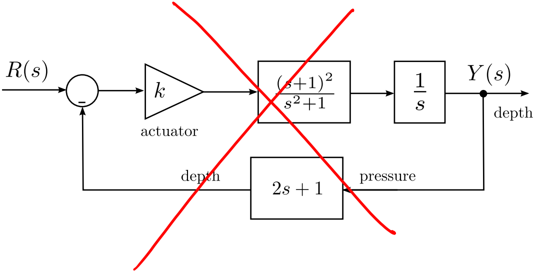
Exercise 139 - solution

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -20 & -29 & -10 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} p$$

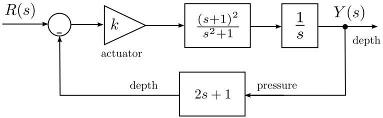
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$

Exercise 140

An automatic depth control system for a robot submarine is shown in the figure. The depth is measured by a pressure transducer. The gain of the stern place actuator is $k = 1$ when the vertical velocity is 25 m/s. Determine a state variable representation of the system.



Exercise 140 - continued

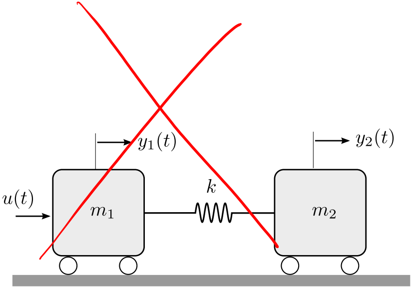


$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1/3 & -5/3 & -5/3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1/3 \end{bmatrix} r$$

$$y = [\,1 \;\; 2 \;\; 1\,]x$$

Exercise 141

A two mass system is shown. The rolling friction constant is b. Determine a state variable representation when the output variable.



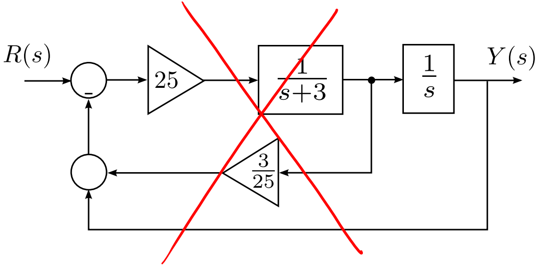
Exercise 141 - solution

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & -\frac{b}{m_1} & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & -\frac{b}{m_2} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [\,0 \;\; 0 \;\; 1 \;\; 0\,]x$$

Exercise 142

A system has block diagram shown. Determine a state variable model.



Exercise 142 - solution

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -25 & -6 \end{bmatrix} x + \begin{bmatrix} 0 \\ 25 \end{bmatrix} r$$

$$y = [\,1 \quad 0\,]x$$

Research opportunities

Students interested in part/full time research in mechatronics with applications to biomedical robotics:

bio
MECHA
TRONICS LAB

www.biomechatronics.ca

The end

Thank you for a great semester!