MECE 3350U
Control Systems

Lecture 14
Implementing PID Controllers

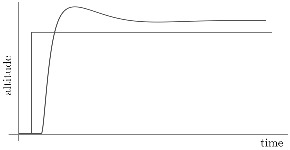
Outline of Lecture 14

By the end of today's lecture you should be able to

- Tune a PID controller
- Understand the limitations of PID controllers
- Understand how to implement PID controllers

Applications

The open-loop step response of the Osprey Tiltrotor aircraft to a step-input is shown in the graph.



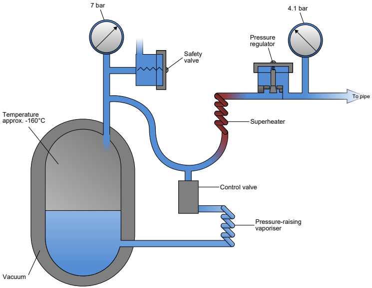
Implement a PID controller to eliminate the steady-state error. How do we select the PID gains?

Does the saturation of the propeller angle affect the performance of the controller?

Applications

A vacuum insulated evaporator allows the bulk storage of cryogenic liquids for industrial and medical applications.

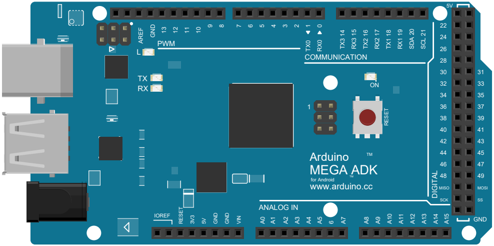
Without functioning insulation, the stored liquid will rapidly warm and undergo a phase transition to gas, increasing in volume and potentially causing a catastrophic failure.



Develop a PID controller to maintain a constant pressure in the vessel without knowing its transfer function.

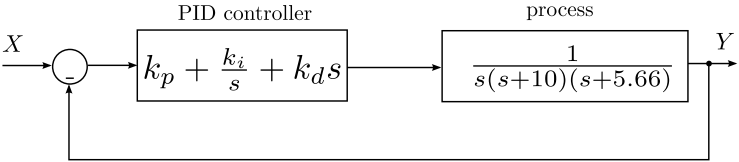
Applications

Can we implement a PID controller using a microcontroller?



PID tuning

Consider the following control scheme. Our object is to find suitable gains for the PID controller.



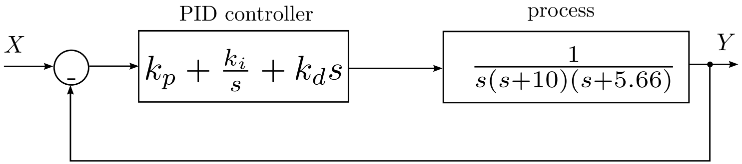
Requirements:

→ Small overshoot (less than 15%)

→ Settling time is less than 3 sec.

→ Zero steady state error.

Manual PID tuning

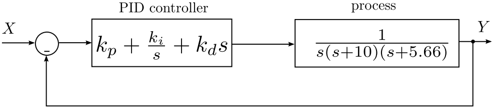


Step 1: Find the critical proportional gain k_p before instability.

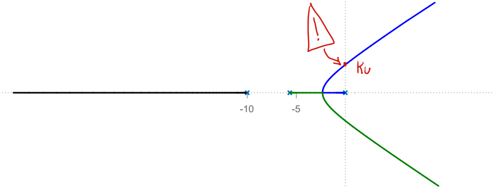
→ Set $k_i = k_d = 0$.

→ Slowly increase k_d to the edge of stability

Manual PID tuning



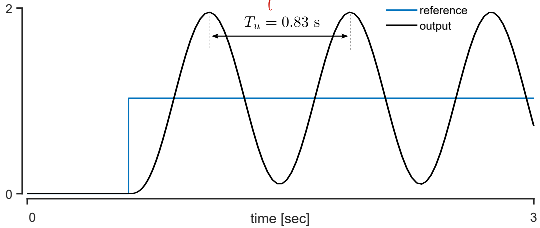
$$1 + k_p \frac{1}{s(s+10)(s+5.66)} = 0$$



How can we calculate the **ultimate gain** k_u?

Manual PID tuning

Ultimate gain: $k_u = 886.356$



Step 2: Reduce k_p to achieve a step response with approximately a quarter amplitude decay.

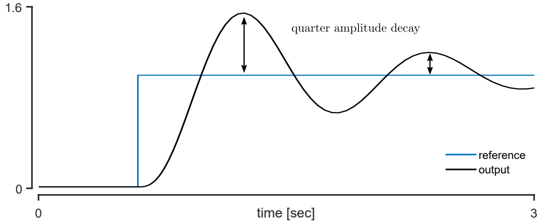
→ I.e,: the overshoot drops to 25% of the initial value after one period.

→ As an initial approximation set $k_p = 0.5k_u$.

→ Period of oscillations $T_u = 0.83$ sec ⇒ **Ultimate period**

Manual PID tuning

Quarter amplitude decay gain: $k_p \approx 0.5k_u = 886.356 \times 0.5$



Step 2: Set the proportional gain and manually analyse the derivative gain.

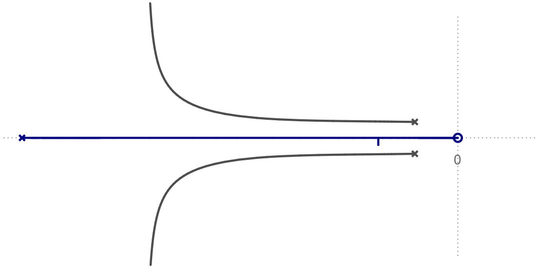
$\rightarrow$ For $k_d > 0$, we have

$$1 + k_d \frac{s}{s(s+10)(s+5.66) + k_p} = 0$$

Manual PID tuning

$k_p = 370,\ k_d > 0,\ k_i = 0$

$$1 + k_d \frac{s}{s(s+10)(s+5.66) + k_p} = 0$$



As k_d increases:

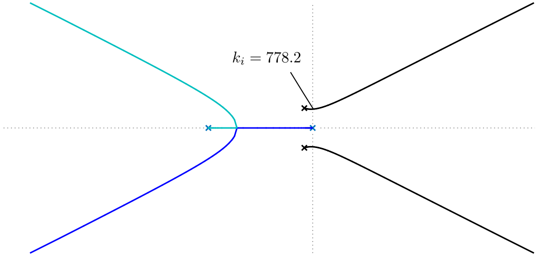
→ Imaginary poles move to the left: Damping ratio increases

→ For large values of k_d, the real pole dominates the response

Manual PID tuning

Step 3: $k_p = 370, \; k_d = 0, \; k_i > 0$

$$1 + k_i \frac{1}{s[s(s+10)(s+5.66) + k_p]} = 0$$



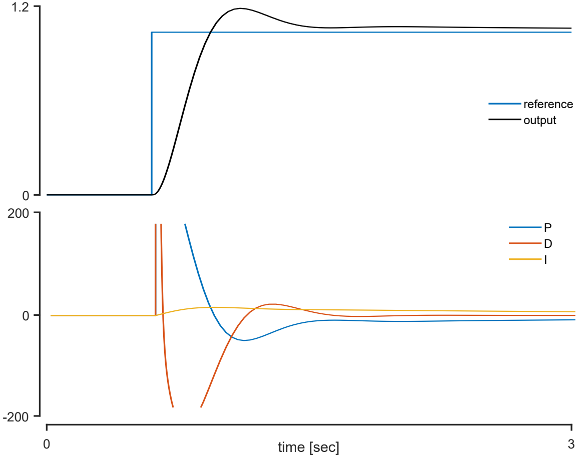
As k_i increases:

→ Complex poles move to the right: Higher overshoot, higher settling time

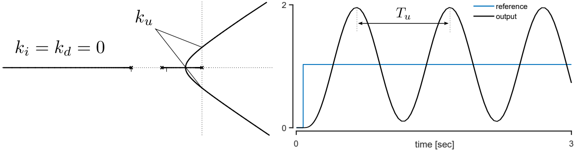
→ What should we do now? Open question since 1936!

Manual PID tuning

Step response for $k_d = 370$, $k_i = 100$, $k_d = 60$.



Ziegler-Nichols PID tuning - Method 1

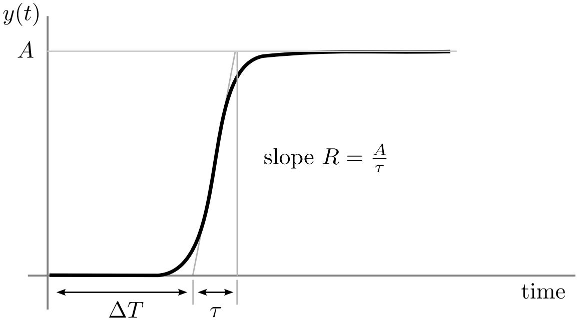


Controller type	k_p	k_i	k_d
Proportional $C(s) = k_p$	$0.5k_u$	0	0
Proportional-integral $C(s) = k_p + k_i s^{-1}$	$0.45k_u$	$\frac{0.54k_u}{T_u}$	0
PID $C(s) = k_p + k_i s^{-1} + k_d s$	$0.6k_u$	$\frac{1.2k_u}{T_u}$	$\frac{0.6K_u T_u}{8}$

Ziegler-Nichols PID tuning - Method 2

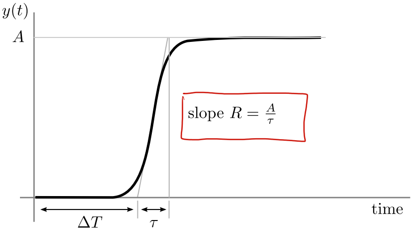
Many systems cane be approximated by the step response of

$$\frac{Y(s)}{U(s)} = \frac{Ae^{-\Delta ts}}{\tau s + 1} \tag{1}$$



This is a first order system with a time delay of Δt sec.

Ziegler-Nichols PID tuning - Method 2



Controller type	k_p	k_i	k_d
Proportional $C(s) = k_p$	$\frac{1}{R\Delta T}$	0	0
Proportional-integral $C(s) = k_p + k_i s^{-1}$	$\frac{0.9}{R\Delta T}$	$\frac{0.27}{R\Delta T^2}$	0
PID $C(s) = k_p + k_i s^{-1} + k_d s$	$\frac{1.2}{R\Delta T}$	$\frac{0.6}{R\Delta T^2}$	$\frac{0.6}{R}$

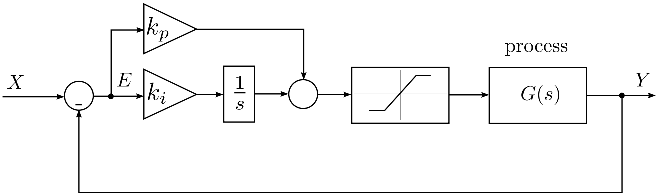
Integrator anti-windup

In many control systems, the output of the actuator can saturate.

→ A valve saturates when it is fully open

→ Control surfaces of an aircraft cannot bend beyond certain angles

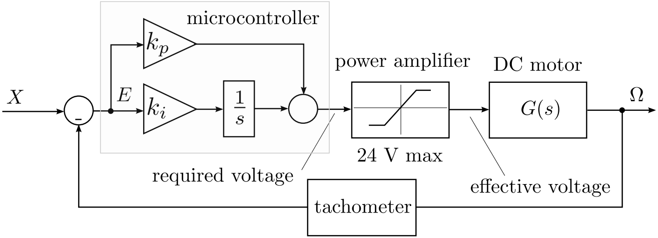
→ The output voltage of a motor speed controller is limited

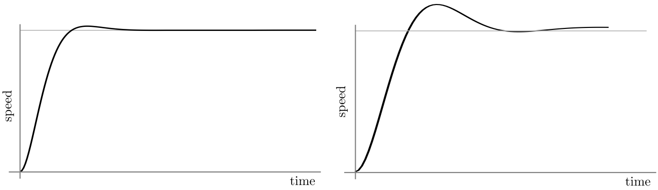


What are the effects of saturation in the controller?

Integrator anti-windup

Example: Consider a PI speed controller for a DC motor.



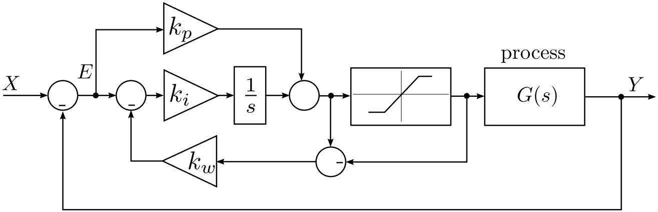


Integrator anti-windup

Solution 1: If the controller is implemented digitally:

$$\text{if } |u| \geq u_{max}, \text{ set } k_i = 0$$

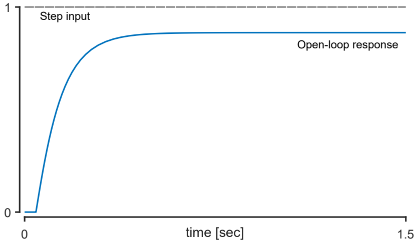
Solution 2: Anti windup loop



k_w can be determined experimentally.

Exercise 68 - Using Matlab

As a control engineer, you were required to design a cruise speed controller for a supersonic aircraft. The dynamic model of the system is unknown. The open-loop response of the aircraft to a step-input signal was measured and is shown in the graphic.



Bonus marks solution will be posted later.

Implement a PID controller and find the optimal gains based on the Ziegler-Nichols tuning method.

Exercise 68 - continued

⇒ Open the Matlab file "PID-tuning-ZNI.m"

⇒ Open the Simulink file "PID-tuning-ZN.slx"

⇒ Run the Matlab script

⇒ Based on the open-loop response, determine the PID gains

⇒ Tune the controller

⇒ Add a small disturbance to the system and compare the open and closed loop response (D = 0.5)

Exercise 69 - Using Matlab

Consider a plant in an unit feedback configuration with the following transfer function for small signals

$$G(s) = \frac{1}{s}$$

and PI controller

$$D(s) = 2 + \frac{4}{s}.$$

Study the effect of windup and antiwindup on the response of the system.

Exercise 69 - continued

⇒ Open the Matlab file "PID-windup-1.m"

⇒ Open the Simulink file "PID-windup.slx"

⇒ Run the Matlab script

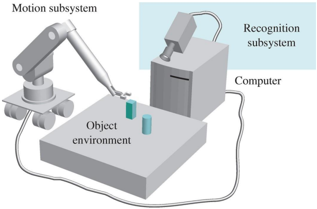
⇒ For kw $= 0$, there is no antiwindup

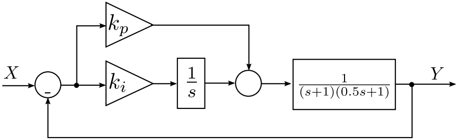
⇒ Increase kw and observe the effect on the overshoot and control effort

Simulink simulation → no solution

Exercise 70 - Using Matlab

A mobile robot using a vision system as the measurement device is shown. Design the controller so that the percent overshoot for a step input is less than 5% and the settling time is less than 6 seconds.





Note: there are several possible solutions to this problem.

Exercise 71 - Using Matlab

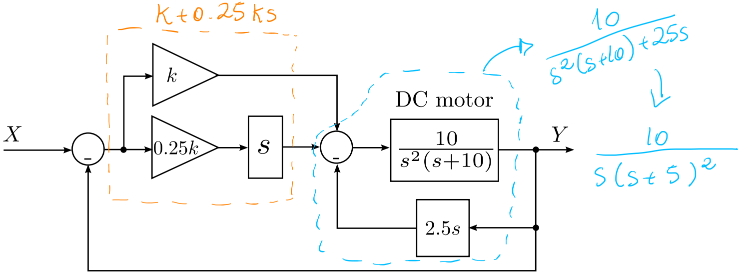
The loop transfer function is $G(s) = \dfrac{K_p s + K_i}{s(s+1)(0.5s+1)}$

One possible set of PI gains is $\begin{cases} K_p = 0.82 \\ K_i = 0.9 \end{cases}$

There are many other solutions.

Exercise 72

A welding torch is remotely controlled to achieve high accuracy while operating in hazardous environments. A model of the arm control is shown.



Select k to provide a satisfactory step response with P.O. $< 5\%$.

Exercise 72 - continued

Recall that P.O. $= 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

P.O. $= 5 \rightarrow \zeta = 0.69$

$1 + K(0.25s+1)\left(\frac{10}{s(s+5)^2}\right)$

factor 2.5

characteristic equation $= 1 + \frac{K(2.5)(s+4)}{s(s+5)^2} = 0$

poles $= -5, -5, 0$

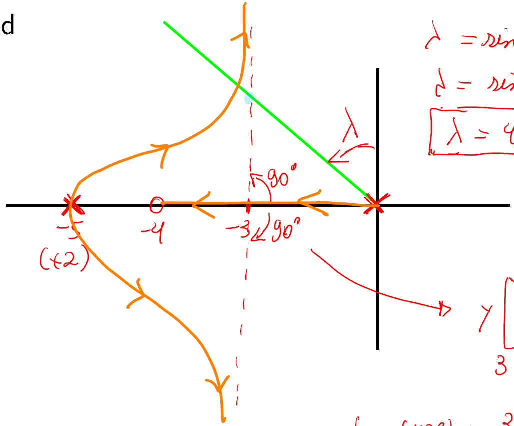
zeros $= -4$

Exercise 72 - continued

$n - m = 2$

$\theta_1 = 90^\circ$

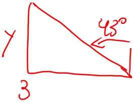
$\theta_2 = -90^\circ$



$\lambda = \sin^{-1}(\zeta)$

$\lambda = \sin^{-1}(0.69)$

$\boxed{\lambda = 43^\circ}$



$\tan(43^\circ) = \frac{3}{Y}$

$\boxed{Y = 3.14}$

$\boxed{s = -3 \pm 3.14j}$

$\alpha = \frac{\sum p - \sum z}{2}$

$\alpha = \frac{-5-5-(-4)}{2} = -3$

for $\zeta = 0.69$

→ pole must be

Exercise 72 - continued

$s = -3 \pm 3.14j$

$$\left|\frac{2.5K(s+4)}{s(s+5)^2}\right| = |1|$$

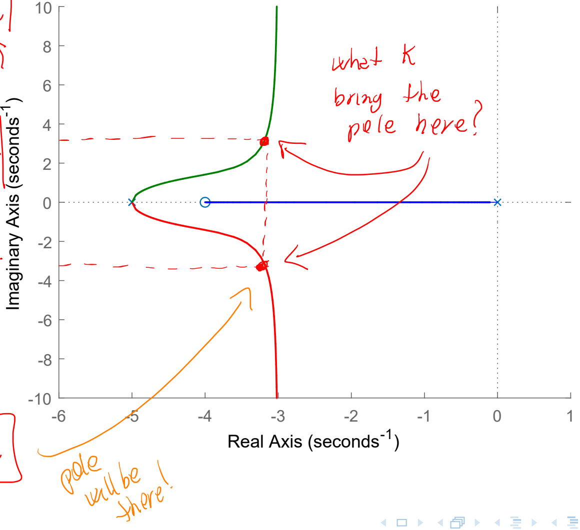
replace $s = -3 \pm 3.14j$

$$2.5K\left|\frac{1+3.14j}{(-3+3.14j)(2+3.14j)}\right|$$

$$\frac{2.5K\sqrt{1+3.14^2}}{\sqrt{9+3.14^2}\left(\sqrt{4+3.14^2}\right)^2} = 1$$

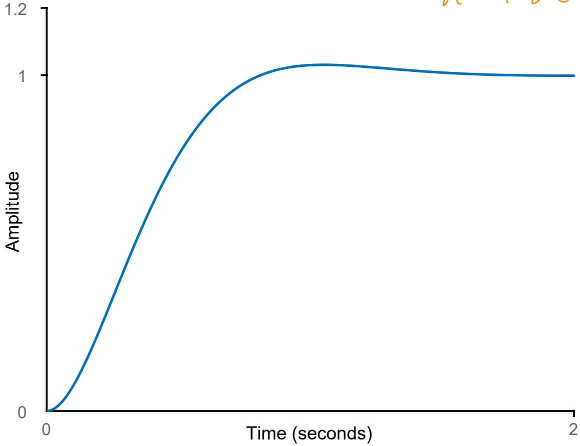
solving for K

$K = 7.32$



Exercise 72 - continued

step response for
K= 7.32.



Next class...

- Midterm review