MECE 3350U
Control Systems

Lecture 13
PID Controllers

Outline of Lecture 13

By the end of today's lecture you should be able to

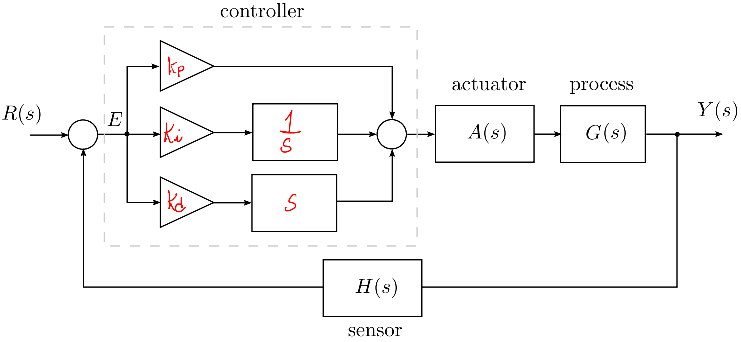
- Define a PID controller
- Understand the influence of the controller in the temporal response
- Understand the practical applications of PID control

Applications

Proportional-Integral-Derivative (PID) controllers are used in most automatic process control applications in industry.

More than 90% of industrial controllers are PID.

PID controller



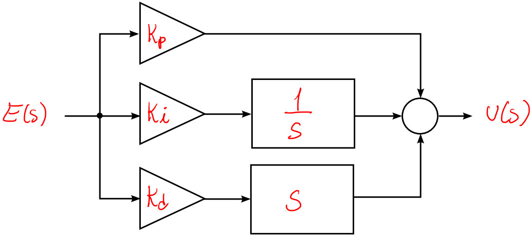
The command signal is a function of

$\rightarrow$ The magnitude of the current error: proportional gain

$\rightarrow$ The integral of the error over time: integral gain

$\rightarrow$ The time rate change of the error: derivative gain

PID controller



$$u(t) = k_p e(t) + k_i \int_0^t e(t)dt + k_d s e(t)$$

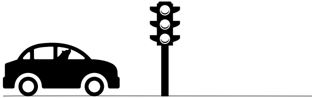
$$U(s) = k_p E(s) + k_i \frac{1}{s} E(s) + k_d s E(s)$$

$$U(s) = \left(k_p + k_i \frac{1}{s} + k_d s\right) E(s)$$

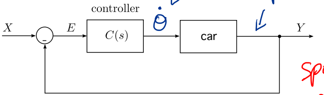
Proportional controller

$G(s) = k_p$, with $k > 0$.

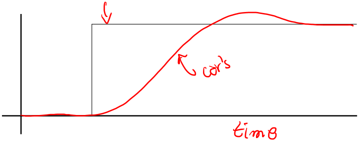
$\dot{\theta}(t) = k_p e(t)$

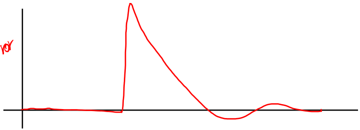


Speed of the pedal

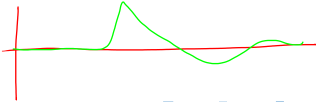


speed control





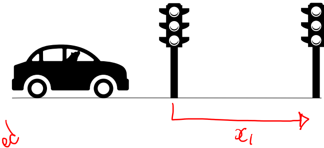
proportional controller → controller output

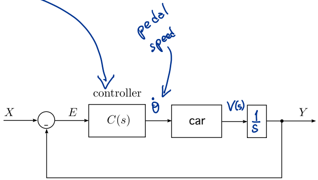


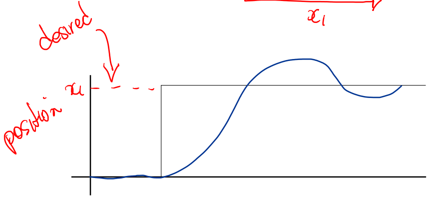
Proportional-derivative (PD) controller

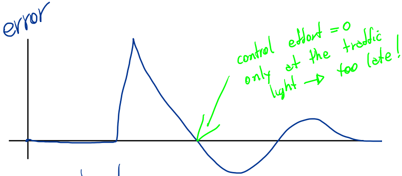
$G(s) = k_p + k_d s$, with $k > 0$.

$\dot{\theta}(t) = k_p e(t) + \underbrace{k_d \frac{d}{dt} e(t)}_{=0}$







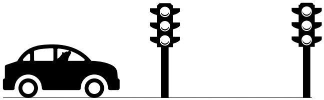


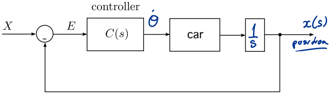
proportional only = overshoot!

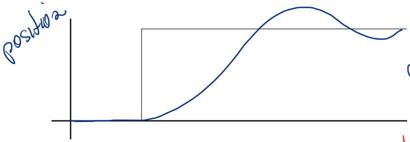
Proportional-derivative (PD) controller

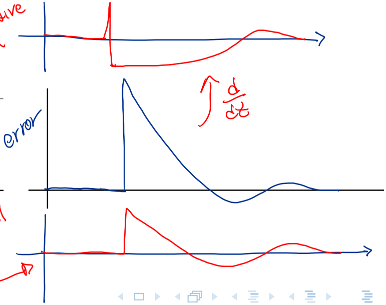
$$G(s) = k_p + k_d s, \text{ with } k > 0.$$

$$\dot{\theta}(t) = k_p e(t) + \underbrace{k_d \frac{d}{dt} e(t)}_{\neq 0}$$









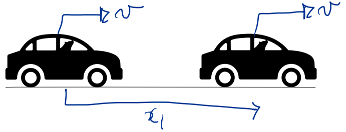
P + D = less overshoot

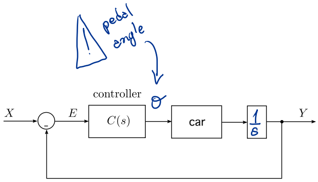
Proportional-integral-derivative (PID) controller

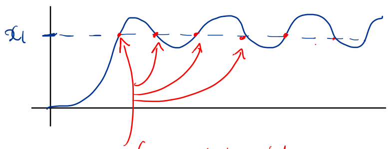
$$G(s) = k_p + k_d s + k_d \frac{1}{s}, \text{ with } k > 0.$$

$$\theta(t) = k_p e(t) + k_d \frac{d}{dt} e(t) + k_i \int e(t)dt$$

objective
$x_1 = 0$



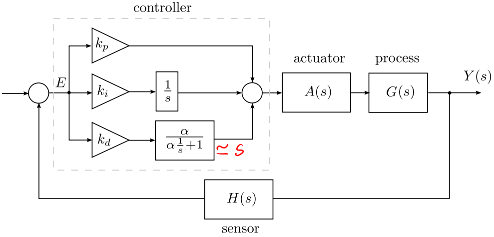




$\theta = 0$! = steady-state error

eliminates steady state error

PID implementation



$$u(t) = k_p e(t) + k_i \int_0^t e(t)dt + k_d \frac{d}{dt} e(t)$$

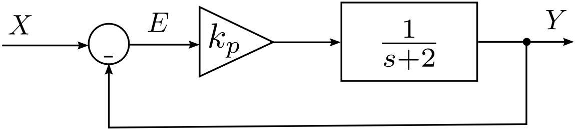
→ Proportional gain: Consider only the current error

→ Integral gain: Looks at the past error

→ Derivative gain: "Anticipates" the future error

PID controller

Effect of k_p



The transfer function is

$$T(s) = \frac{k_p}{s + k_p + 2} \tag{1}$$

The time constant is $1/(k_p + 2)$.

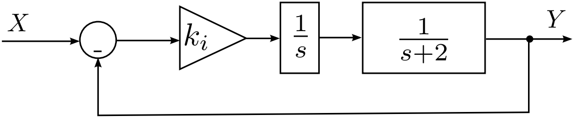
The steady-state error for $x(t) = 1$ is

Kp reduces e(∞)

$$e(\infty) = u(t) - \lim_{s\to 0}[sF(s)] = 1 - \lim_{s\to 0} s\frac{k_p}{s + k_p + 2}\frac{1}{s} = 1 - \frac{k_p}{k_p + 2} \tag{2}$$

PID controller

Effect of the integrator and k_i on the transient response



The transfer function is

$$T(s) = \frac{k_i}{s(s+2)+k_i} = \frac{k_i}{s^2+2s+k_i} \qquad (3)$$

where $\omega_n = \sqrt{k_i}$ and $\zeta = 1/\sqrt{k_i}$.

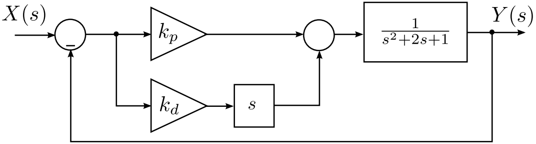
The **damping ratio decreases** with k_i.

Overshoot: $P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 100e^{-\pi/\sqrt{k_i-1}}$

$\boxed{\text{The \textbf{overshoot increases} with } k_i}$

PID controller

Effect of the derivative gain k_d



The transfer function is

$$T(s) = \frac{sk_d + k_p}{s^2 + (2 + k_d)s + 1 + k_p} \tag{4}$$

note the final value does not depend on k_d, thus it has **no effect** on the steady-state error.

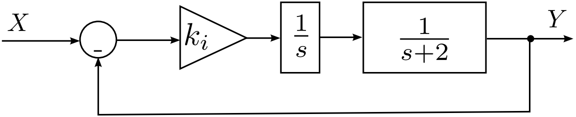
From the characteristic equation: $\zeta = (2 + k_d)/2(\sqrt{1 + k_p})$, $\omega_n = \sqrt{1 + k_p}$

The overshoot: $P.O. = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$ **decreases** with k_d.

The settling time: $Ts = 4\omega_n\zeta$ **decreases** with k_d.

PID controller

Effect of the integrator on the steady-state error



The transfer function is

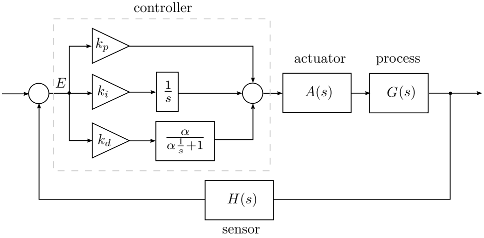
$$T(s) = \frac{k_i}{s(s+2)+k_i} = \frac{k_i}{s^2+2s+k_i} \tag{5}$$

Steady-state error

$$e(\infty) = u(t) - \lim_{s\to 0}[sF(s)] = 1 - \lim_{s\to 0} s\frac{k_i}{s(s+2)+k_i}\frac{1}{s} = 0 \tag{6}$$

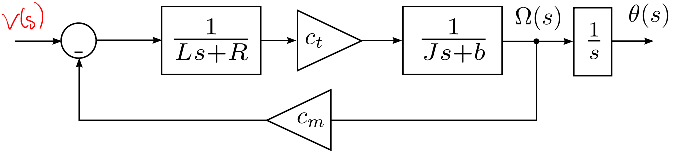
always $e(\infty) = 0$ if $K_i \neq 0$

PID gains



PID gain	Overshoot	Settling time	Steady-state error
Increasing k_p	Increases	Minimal impact	Decreases
Increasing k_i	Increases	Increases	Zero error
Increasing k_d	Decreases	Decreases	No impact

Speed control of a DC motor

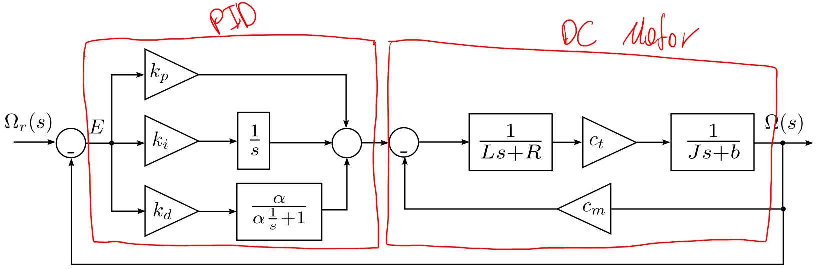


DC motor characteristics:

$J = 1.13 \times 10^{-2} Nm - sec^2/rad,\ b = 0.028 Nm - sec/rad,\ L = 0.1$ H

$R = 0.45$ Ohms, $c_t = 0.067$ Nm/A, $c_m = 0.067$ V-sec/rad

Speed control of a DC motor



$\alpha = 100$

See Simulink model posted on B.B.

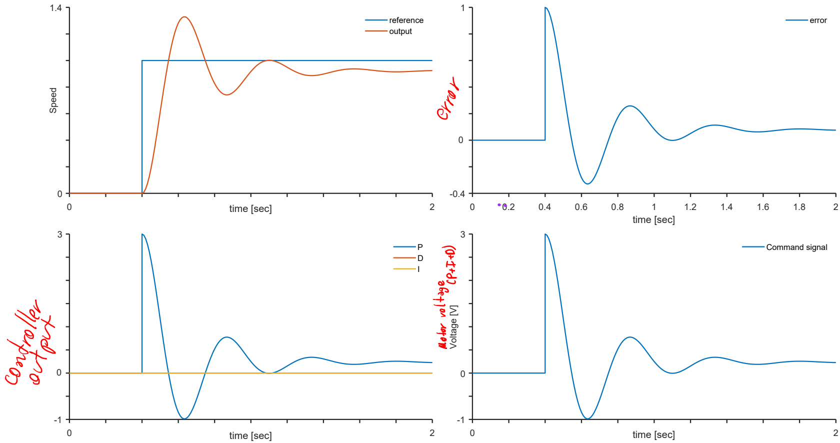
Proportional control: $k_p = 3,\ k_d = 0,\ k_i = 0$

Proportional-integral control: $k_p = 3,\ k_d = 15,\ k_d = 0$

PID control: $k_p = 3,\ k_d = 15,\ k_d = 0.3$

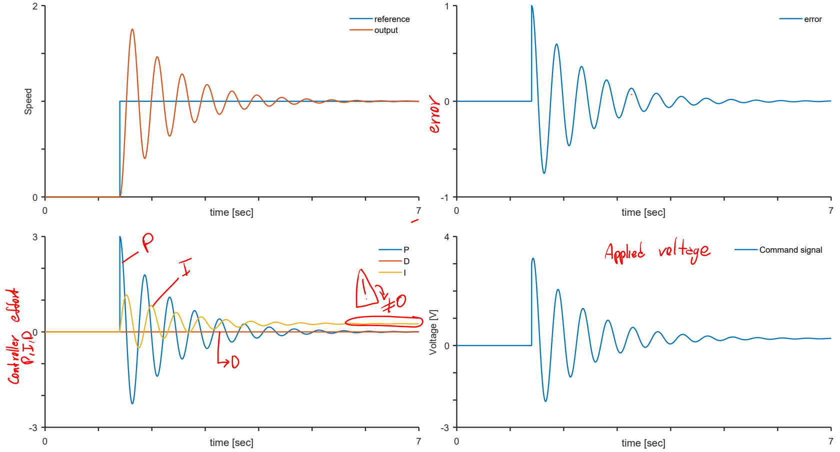
Speed control of a DC motor

Proportional control: $k_p = 3, \ k_d = 0, \ k_i = 0$



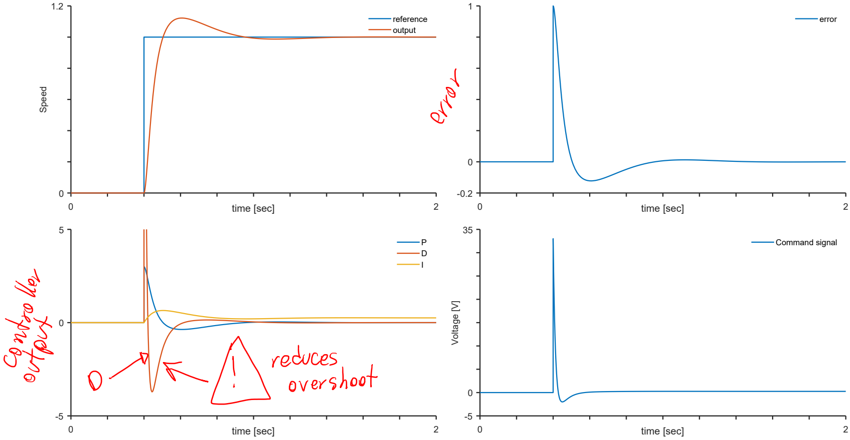
Speed control of a DC motor

Proportional-integral control: $k_p = 3$, $k_i = 15$, $k_d = 0$

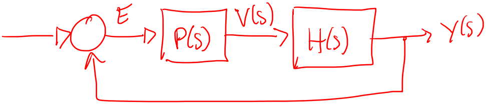


Speed control of a DC motor

Proportional-integral-derivative control: $k_p = 3,\ k_d = 15,\ k_i = 0.3$



Exercise 66



A block diagram of a diesel engine driving load has an transfer function connecting the fuel valve setting $V(s)$ and the load shaft speed $Y(s)$ given by

$$H(s) = \frac{Y(s)}{V(s)} = \frac{1}{0.01s^2 + 0.11s + 0.1}. \tag{7}$$

Speed control is provided by a PID controlled whose transfer function is

$$P(s) = \frac{V(s)}{E(s)} = 5 + 0.3s + \frac{5}{sT} \tag{8}$$

By using the root-locus method, determine the minimum value τ if the complex closed loop dominant poles are to have a damping ratio not less than 0.7.

(1) write the root-locus function as $1 + \tau Z(s) = 0$ $Z(s)$ is a function of s

Exercise 66 - continued

zeros $\rightarrow \begin{cases} s=0 \\ s=-20.5 \pm 9.5i \end{cases}$

$n-m=3$

$g=1 \quad \theta=60^\circ$

$g=2 \quad \theta=180^\circ$

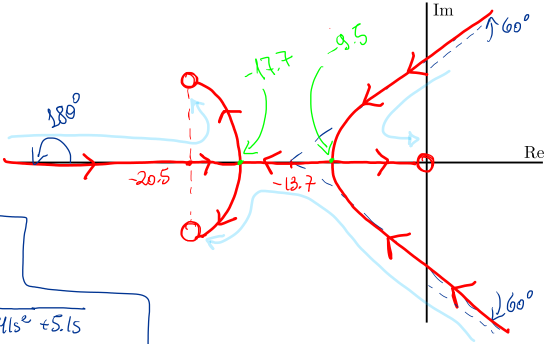
$g=3 \quad \theta=-60^\circ$

$$1+\tau\left(\frac{0.01s^3+0.41s^2+5.1s}{5}\right)=0 \qquad (9)$$

$\alpha = \dfrac{\sum p - \sum z}{3}$

$\alpha = \dfrac{-20.5-20.5-0}{3}$

$\boxed{\alpha = -13.7}$



Break away/in points

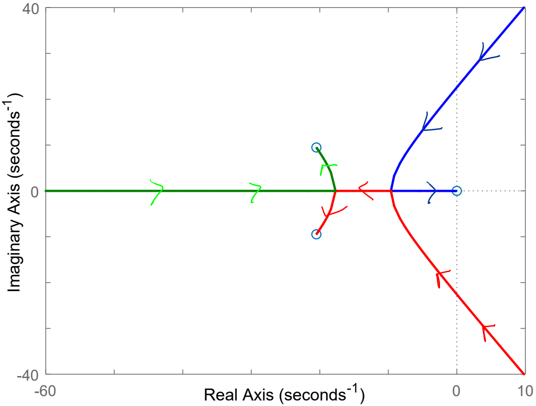
$\tau = p(s)$

$p(s) = -\dfrac{5}{0.01s^3+0.41s^2+5.1s}$

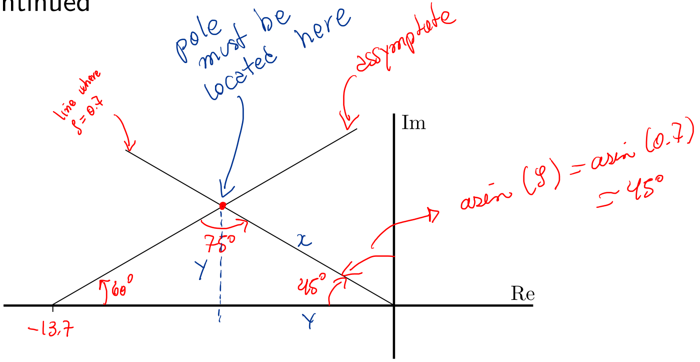
$\dfrac{dp(s)}{ds}=0 \rightarrow \begin{cases} s=-9.5 \\ s=-17.7 \end{cases}$

Exercise 66 - continued

$$1 + \tau \left(\frac{0.01s^3 + 0.41s^2 + 5.1s}{5} \right) = 0 \tag{10}$$



Exercise 66 - continued



$$\frac{\sin(75^\circ)}{13.7} = \frac{\sin(60^\circ)}{x}$$

$$\boxed{x = 12.2}$$

$$Y = x \sin(45^\circ)$$

$$\boxed{Y = 8.7}$$

pole is

$$\boxed{s = -8.7 \pm 8.7j}$$

Exercise 66 - continued

$s = -8.7 \pm 8.7j$ replace s here, find τ

$$(0.01s^3 + 0.11s^2 + 0.1s)\tau + (0.3s^2 + 5s)\tau + 5 = 0$$

$$\boxed{\tau = 0.16}$$

Centre of the asymptote is constant. $\forall \tau$

Thus we can estimate the location of the 3 pole as

$$-13.7 = \frac{\sum p - \sum z}{3}$$

$$-13.7 = \frac{-8.7 - 8.7 - p_2}{3}$$

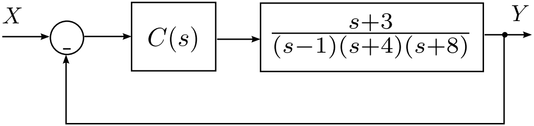
$$\boxed{p_2 = -23.6}$$

(insignificant pole)

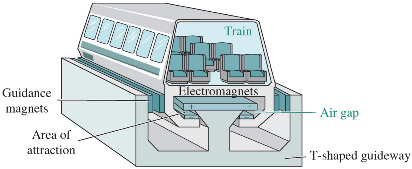
Exercise 67

Kd=0

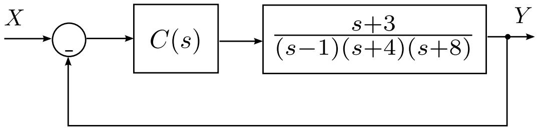
The air gap control system in a magnetically levitated high-speed train is controlled by a *PI controller* $G(s)$ have the same k_i and k_p gains.



Select the controller gain $k = k_i = k_p$ so that all of the complex poles have a damping ratio higher than 0.6.



Exercise 67 - continued



$K_d = K_i = K$

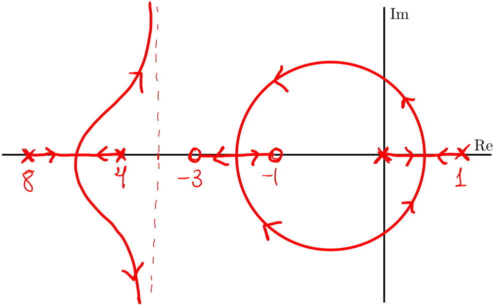
$$C(s) = \frac{K}{s} + K = K\left(\frac{1+s}{s}\right)$$

$$1 + K\frac{(s+1)(s+3)}{s(s+4)(s+8)(s-1)} = 0$$

Exercise 67 - continued

$$1 + k\frac{(s+1)(s+3)}{s(s-1)(s+4)(s+8)} = 0 \qquad (11)$$

$n - m = 2$

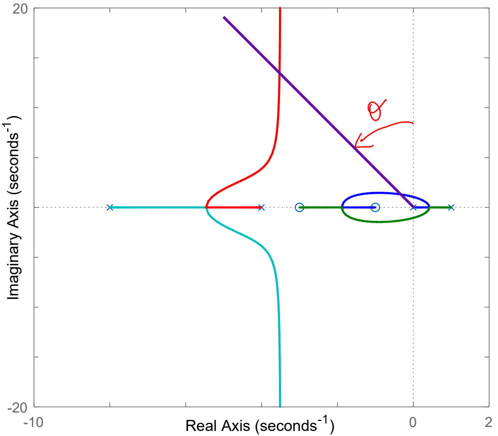


Asymptotes: $2, \ 90^0, \ -90^0$

Centroid: $\alpha = \dfrac{(0-4-8+1)-(-1-3)}{2} = -3.5$

Exercise 67 - continued

$$1 + k\frac{(s+1)(s+3)}{s(s-1)(s+4)(s+8)} = 0 \qquad (12)$$

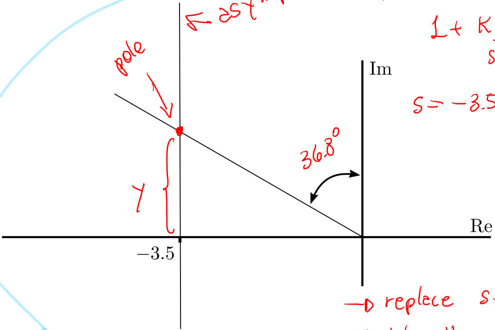


$\theta = \sin^{-1}(0.6)$

desired damping

$\theta = 36.8°$

Exercise 67 - continued



to find K:

$$1+\frac{K(s+1)(s+3)}{s(s+1)(s+4)(s+8)}=0$$

$s=-3.5+4.66j$ replace.

→ replace $s=-3.5+4.66j$

calculate the magnitude at $= 1$

$$1 = \underbrace{K\,\frac{\sqrt{(-3.5+1)^2+4.66^2}\,\sqrt{(-3.5+3)^2+4.66^2}}{\sqrt{3.5^2+4.66^2}\,\sqrt{(-3.5+4)^2+4.66^2}\,\sqrt{(-3.5+1)^2+4.66^2}\,\sqrt{5.5^2+4.66^2}}}_{=1}$$

$$\boxed{K=46.1}$$

$$\tan(36.8^\circ)=\frac{3.5}{Y}$$

$$\boxed{Y=4.66}$$

pole is $\boxed{s=-3.5\pm4.66j}$

Next class...

- Tuning a PID controller