METE 3100U
Actuators and Power Electronics

Lecture 7

Pulse Width Modulation

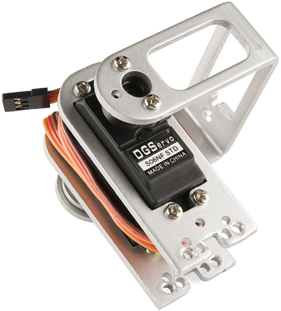
Outline of Lecture 7

By the end of today's lecture, you should be able to

- Understand the principles of pulse width modulation
- Control the voltage of a single phase inverter using PWM
- Model an PWM controlled circuit

Applications

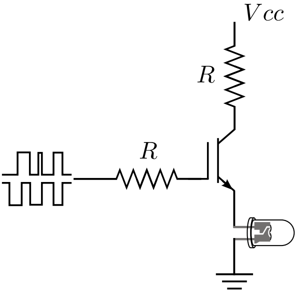
Pulse width modulation is widely used to control the angle of a servo motor attached to a robot arm.



https://learn.sparkfun.com/tutorials/pulse-width-modulation/all

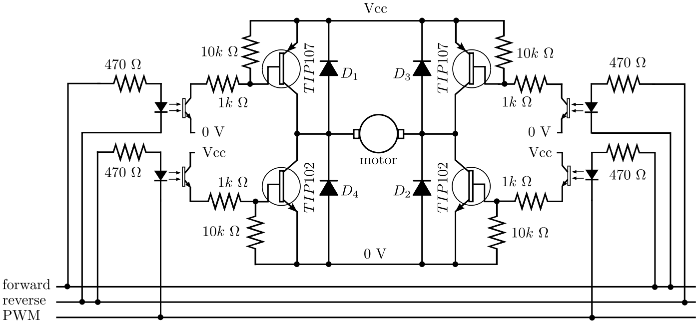
Applications

By controlling the duty cycle of the input signal, the voltage applied to the LED can be changed.



Applications

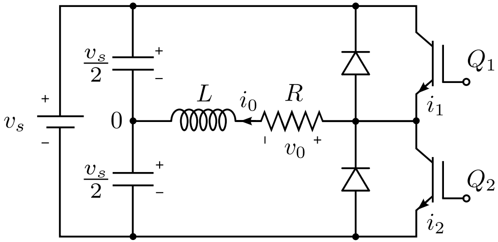
How can the full bridge inverter be controlled in such a way that the motor follows a specific velocity profile?



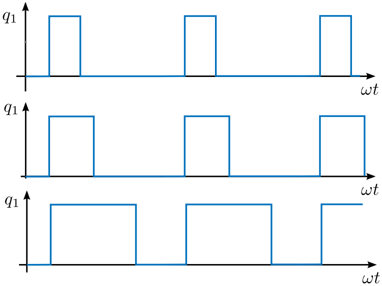
`https://www.electronics-tutorials.ws/blog/pulse-width-modulation.html`

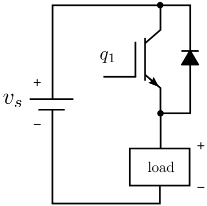
Applications

If we wish to apply a sinusoidal voltage to the load, how should the switches Q_1 and Q_2 be timed?



DC/DC converter duty cycle





The average voltage across the load is

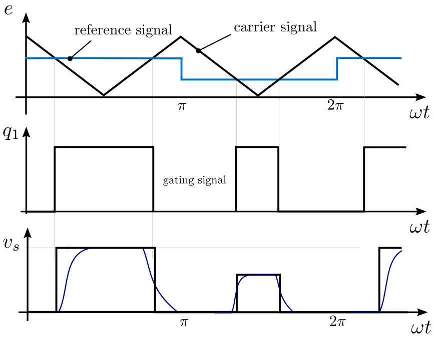
$$v_0 = v_s k \tag{1}$$

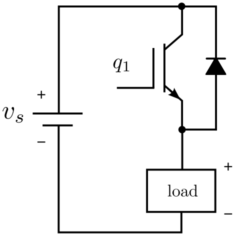
The rsm voltage across the load is

$$v_{0r} = v_s \sqrt{k} \tag{2}$$

Pulse width modulation

Working principle: Compare a reference signal with a triangular carrier ware



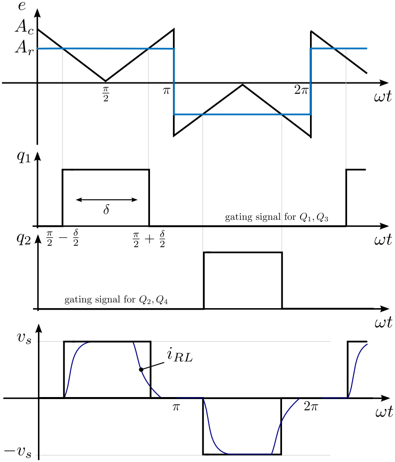


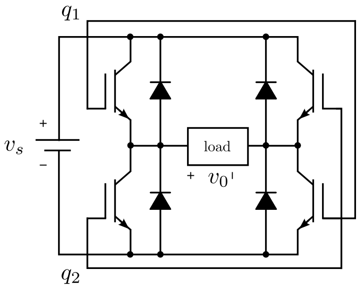
Reference signal: Magnitude A_r

Carrier signal: Magnitude A_c

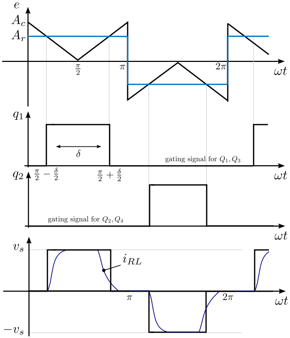
Single pulse width modulation

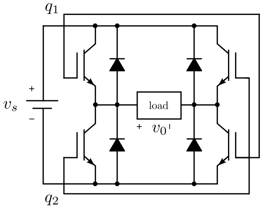
Only one pulse per half cycle exists. The pulse width is varied to control the output voltage.





Single pulse width modulation





The controlled variable is the modulation index M

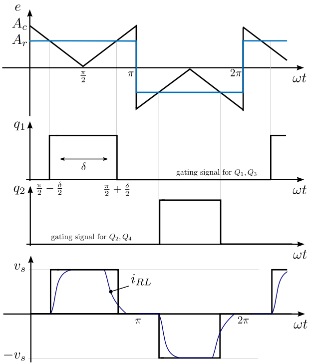
$$M = \frac{A_r}{A_c} = k \tag{3}$$

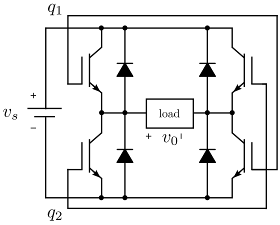
⇒ If $M = 0$, then $v_0 =$

⇒ If $M \geq 1$, then $v_0 =$

For single pulse modulation, $k = M$.

Single pulse width modulation



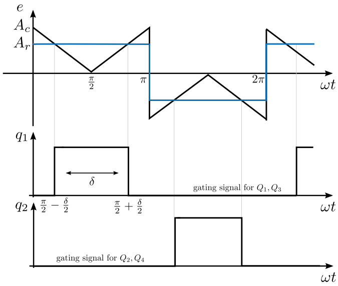


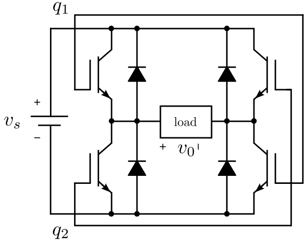
The rms output voltage is

$$V_0 = \sqrt{\frac{2}{2\pi}\int_{\frac{\pi}{2}-\frac{\delta}{2}}^{\frac{\pi}{2}+\frac{\delta}{2}} v_s^2 d(\omega t)} = v_s\sqrt{\frac{\delta}{\pi}} \tag{4}$$

δ is the pulse with in degrees.

Single pulse width modulation





The relation between the pulse with and the modulation index is

$$M = \frac{\delta}{\pi} = \frac{A_r}{A_c} \tag{5}$$

Thus, the rms output voltage is

$$V_0 = v_s\sqrt{M} \tag{6}$$

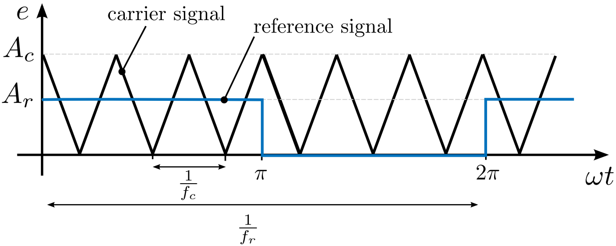
Multiple pulse width modulation (MPWM)

Working principle: Compare a reference signal with a triangular carrier ware

Carrier signal: Frequency f_c, peak value A_c

Reference signal: Frequency f_r, peak value A_r ($f_c >> f_r$)

f_r determines fundamental frequency of the output voltage.

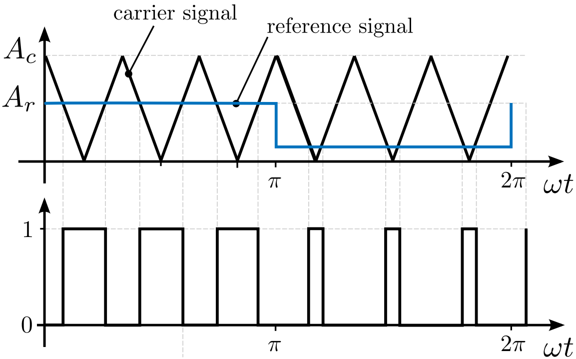


The frequency modulation ratio

$$m_f = \frac{f_c}{f_r} \tag{7}$$

Multiple pulse width modulation

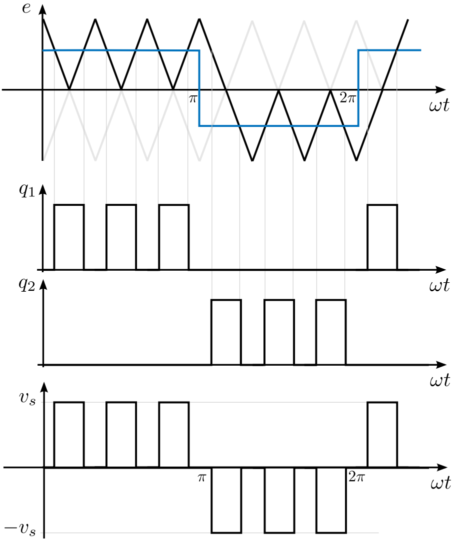
Reference higher than the carrier: the gating signal is on.



The number of pulses per half cycle is

$$p = \frac{f_c}{2f_r} = \frac{m_f}{2} \tag{8}$$

Multiple pulse width modulation

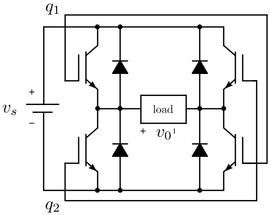


q_1: Signal fed to Q_1 and Q_3

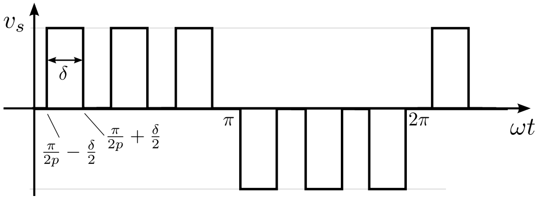
q_2: Signal fed to Q_2 and Q_4

The instantaneous voltage is

$$v_0 = v_s(q_1 - q_2)$$



Multiple pulse width modulation



The rsm output voltage is

$$V_0 = \sqrt{\frac{2p}{2\pi} \int_{\frac{\pi}{2p}-\frac{\delta}{2}}^{\frac{\pi}{2p}+\frac{\delta}{2}} v_s^2 d(\omega t)} = v_s \sqrt{\frac{p\delta}{\pi}} \tag{9}$$

The pulse width is

$$\delta = \frac{M\pi}{p} \tag{10}$$

Thus, the rms voltage is

$$V_0 = v_s \sqrt{M} \tag{11}$$

Multiple pulse width modulation

The Fourier series for the output voltage is

$$v_0(t) = \sum_{n=1,3,5,\ldots}^{\infty} B_n \sin(n\omega t) \tag{12}$$

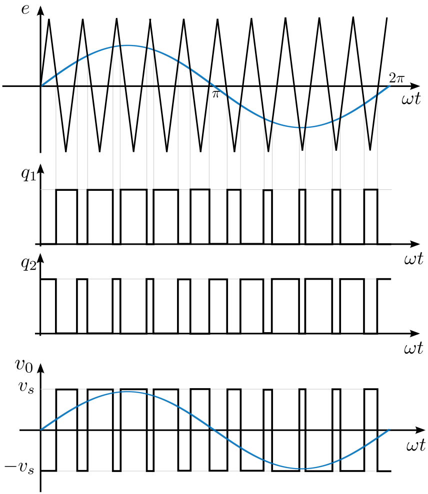
$$b_n = \frac{2}{\pi}\left[\int_{\alpha_m}^{\alpha_m+\delta} \sin(n\omega t)d(\omega t) - \int_{\pi+\alpha_n}^{\pi+\alpha_m+\delta} \sin(n\omega t)d(\omega t)\right]$$
$$b_n = \frac{4v_s}{n\pi}\sin\frac{n\delta}{2}\left[\sin n\left(\alpha_m \frac{\delta}{2}\right)\right]$$

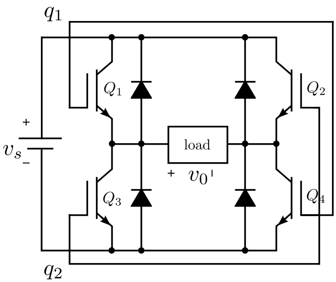
The coefficients B_n can be found by adding the effect of all pulses

$$B_n = \sum_{m=1}^{2p} \frac{4v_s}{n\pi}\sin\frac{n\delta}{2}\left[\sin n\left(\alpha_m + \frac{\delta}{2}\right)\right] \tag{13}$$

Bipolar sinusoidal pulse width modulation

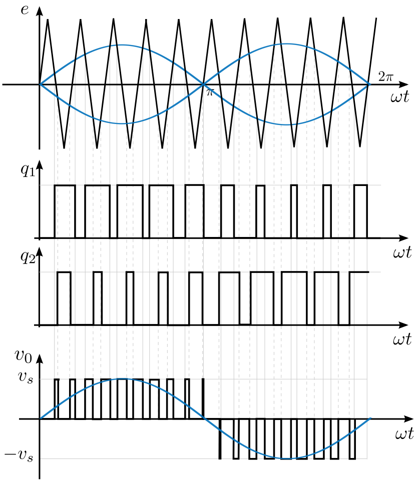
The upper and lower switches in the same leg work in a complementary manner.

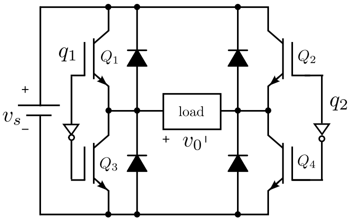




Unipolar sinusoidal pulse width modulation

Two modulated waves 180° out of phase are compared to the carrier signal.





Unipolar *vs* bipolar SPWM

Bipolar modulation

→ During a half cycle, v_0 pulses can be positive and negative

→ All transistors switch at the same time

→ Easier to implement

→ Requires large filter at the output

Unipolar modulation

→ During a half cycle, v_0 pulses can be either positive or negative

→ Transistors do not switch at the same time

→ More efficient than bipolar PWM

→ The distortion factor is improved

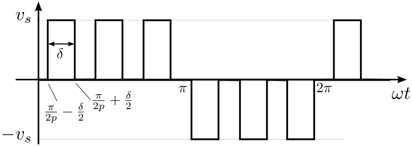
Sinusoidal pulse width modulation

The width of the k^{th} pulse is approximately

$$\delta_k = \frac{\pi}{p} M \sin(\alpha_k)$$

Where α can be obtained by numerically solving

$$M \sin(\alpha_k) = -\frac{2p}{\pi}\alpha_k + 2k - 1 \tag{14}$$



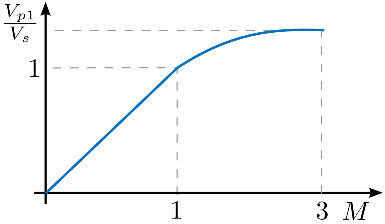
The rms output voltage is the average area under each pulse

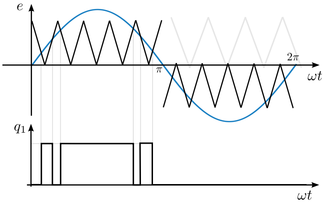
$$V_0 = v_s \sqrt{\sum_{k=1}^{2p} \frac{\delta_k}{\pi}} \tag{15}$$

Under and over modulation

The peak fundamental output voltage is approximately

$$V_{p1} \approx M v_s \quad \forall\, 0 \leq M \leq 1 \tag{16}$$



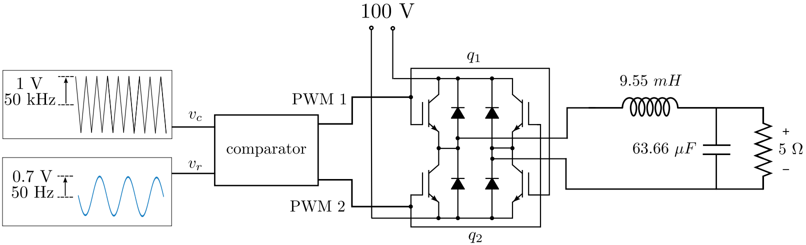


$M \leq 1$ Under-modulation

$M \geq 1$ Overmodulation

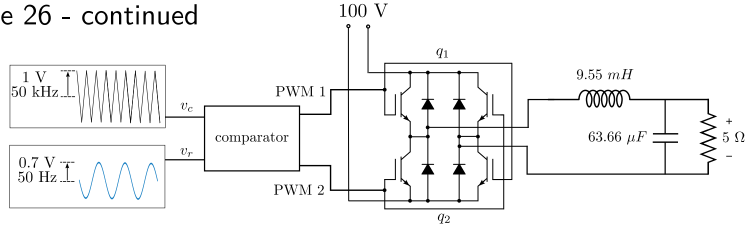
Exercise 26

The full bridge inverter shown uses multiple pulse width modulation to control the output voltage across the 5 Ω resistance.

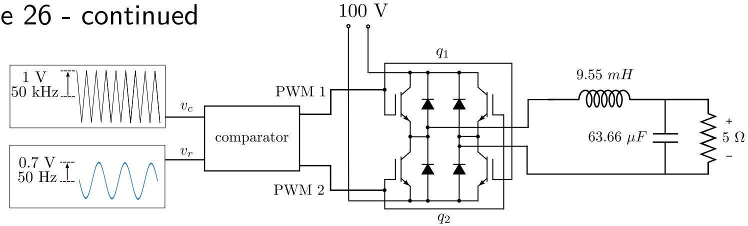


Calculate the **amplitude** of the fundamental component of the output voltage across the resistor.

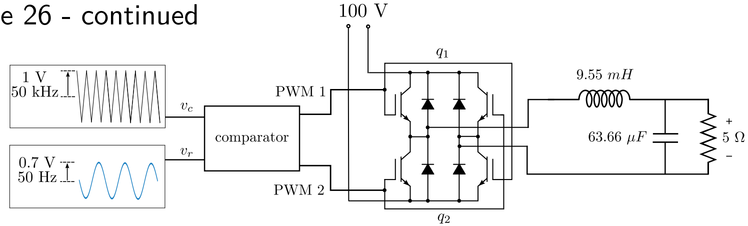
Exercise 26 - continued



Exercise 26 - continued

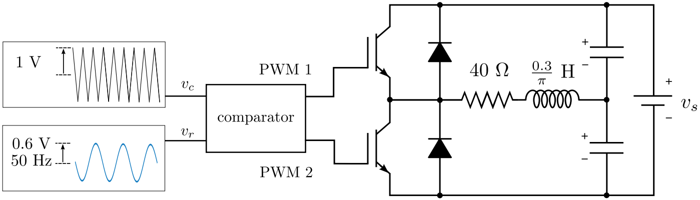


Exercise 26 - continued



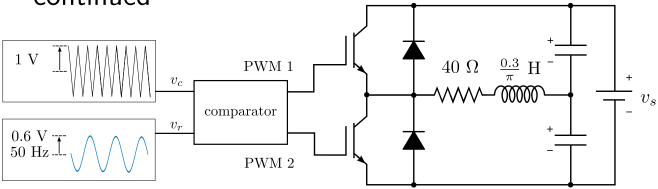
Exercise 27

The half-bridge inverter supplies a RL-load. The control signals are generated using SPWM. At 50 Hz, the load draws an active power of 1.44 kW.

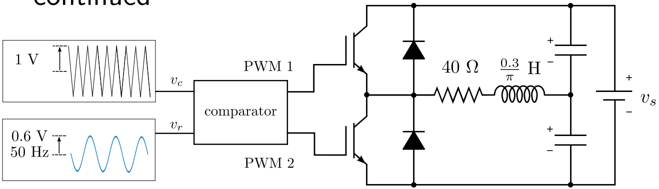


Calculate the value of the DC supply voltage v_s.

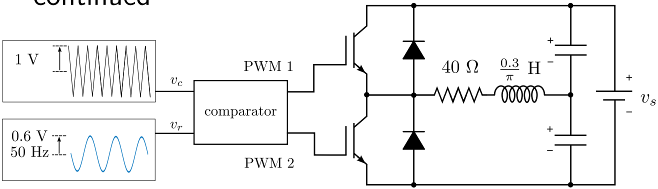
Exercise 27 - continued



Exercise 27 - continued

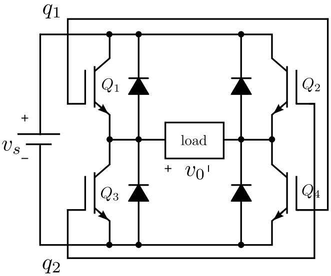


Exercise 27 - continued



Exercise 28

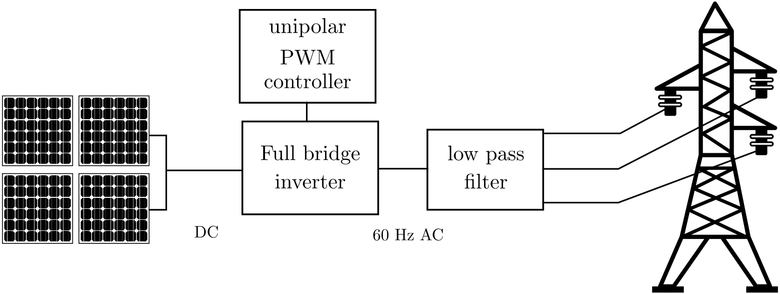
A single phase full bridge inverter uses a uniform PWM with 5 pulses per half cycle. Determine the pulse width if the rms output voltage is 80% of the DC input voltage.



Exercise 28 - continued

Exercise 29a

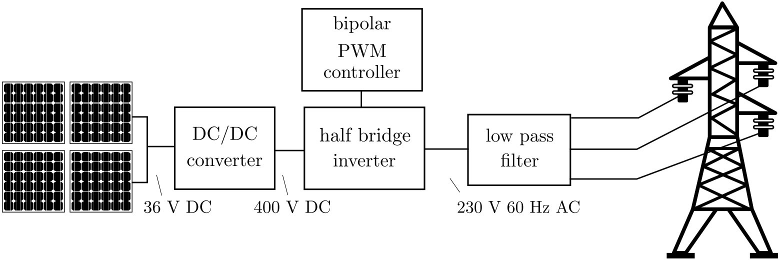
The system shown is designed to convert the 36 V DC voltage from the solar panel to the power line level 60 Hz AC voltage using bipolar.



Implement a Simulink model of the system including a low pass filter for harmonic distortion reduction and analyse the influence of the frequency and amplitude modulation indexes in the output voltage. Assume a RL load.

Exercise 29b

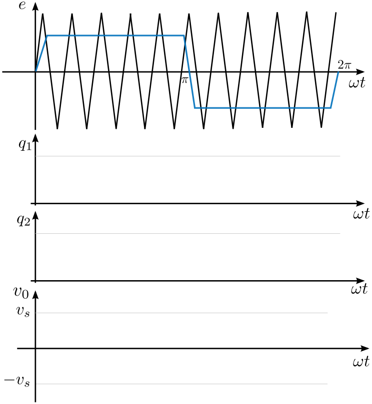
The system shown is designed to convert the 36 V DC voltage from the solar panel to the power line level 230 V, 60 Hz AC voltage using unipolar PWM.



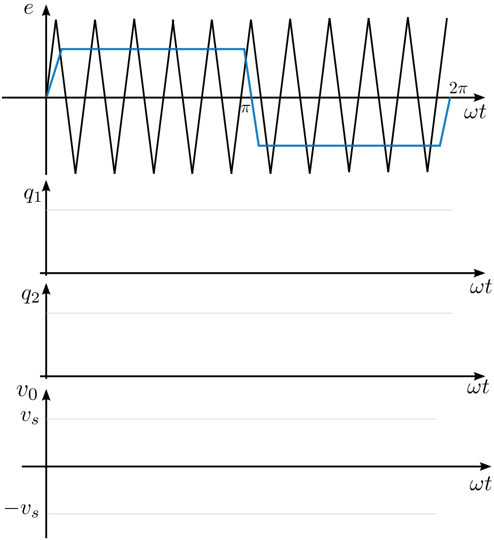
Implement a Simulink model of the system including a low pass filter for harmonic distortion reduction and analyse the influence of the frequency and amplitude modulation indexes in the output current. Assume a RL load.

Exercise 30

A half-bridge inverter with a DC voltage v_s is used to generate a trapezoidal voltage as shown. Draw the waveform of the gating signals and output voltage.

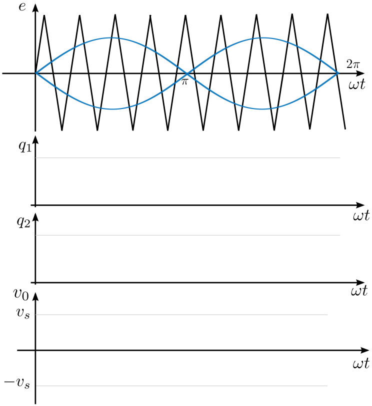


Exercise 30 - continued

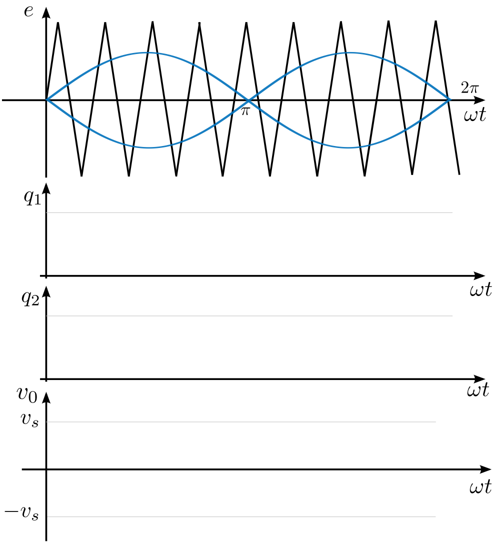


Exercise 31

A half-bridge inverter with a DC voltage v_s uses unipolar PWM to generate a sinusoidal voltage as shown. Draw the waveform of the gating signals and output voltage.

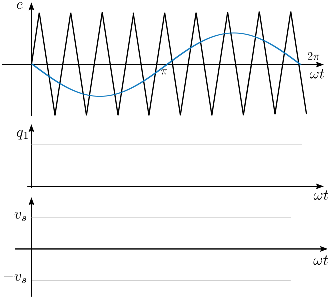


Exercise 31 - continued

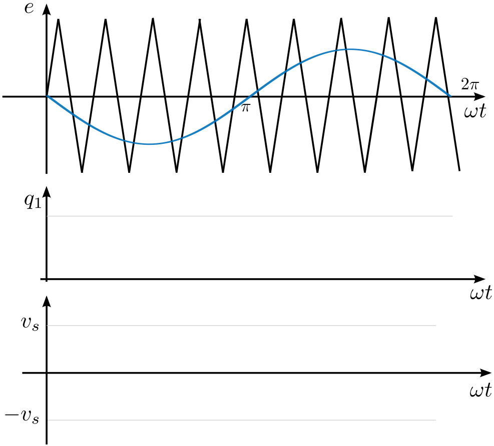


Exercise 32

A half-bridge inverter with a DC voltage v_s uses bipolar PWM to generate a sinusoidal voltage as shown. Draw the waveform of the gating signals and output voltage.



Exercise 32 - continued



Next class...

- 3-phase PWM

Additional supporting materials for Lecture 7:

PWM in Arduino: `https://goo.gl/ch1k6C`

Solar panel voltage control using PWM: `https://goo.gl/D7xPpn`

H-bridge circuit analysis: `https://goo.gl/gGTKzb`