METE 3100U
Actuators and Power Electronics

Lecture 6
DC/AC Converters

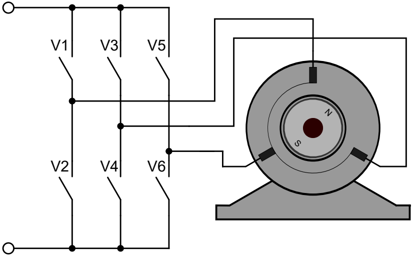
Outline of Lecture 6

In today's lecture we will

- Understand the principles of DC/AC converters
- Analyse and design DC/AC converters
- Understand the principles of voltage modulation

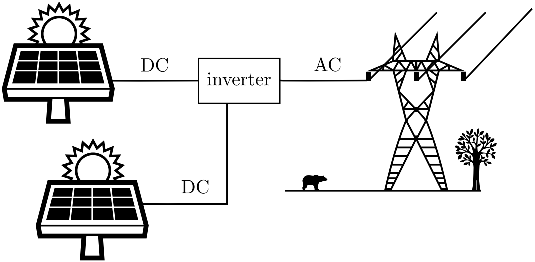
Applications

Inverter circuits designed to produce a variable output voltage range are often used within motor speed controllers.

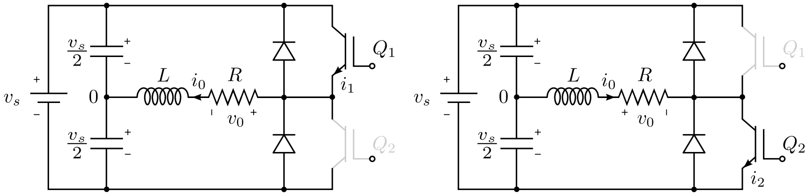


Applications

A solar inverter converts the DC output of a solar panel into a utility frequency AC current that can be fed into a commercial electrical grid.



Half bridge inverter



Q_1 on, Q_2 off: the voltage across the load is

$$v_0 = \tag{1}$$

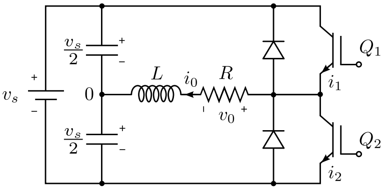
Q_2 on, Q_1 off: the voltage across the load is

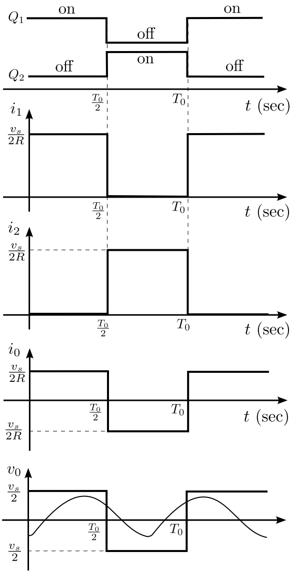
$$v_0 = \tag{2}$$

Q_1 and Q_2 on: the voltage across the load is

$$v_0 = \tag{3}$$

DC/AC converters





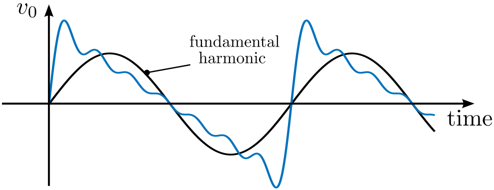
The root mean square of the output voltage is

$$v_0 = \sqrt{\frac{1}{T}\int_0^T v_0^2 dt}$$

$$v_0 = \sqrt{\frac{1}{T}\int_0^T \left(\frac{v_s}{2}\right)^2 dt} = \frac{v_s}{2}$$

Fundamental harmonics

A harmonic is a voltage or current at a multiple of the fundamental frequency, produced by the action of non-linear loads such as voltage converters.



Using a Fourier series, the load voltage is

$$v_0 = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right] \tag{4}$$

Due to symmetry around the x-axis, $a_0 = a_n = 0$.

Fundamental harmonics

We get b_n as

$$b_n = \frac{1}{n}\left[\int_{-\frac{\pi}{2}}^{0} -\frac{v_s}{2}\sin(n\omega t)d(\omega t) + \int_{0}^{\frac{\pi}{2}} \frac{v_s}{2}\sin(n\omega t)d(\omega t)\right] = \frac{2v_s}{n\pi} \quad (5)$$

Thus the instantaneous output voltage is

$$v_0 = \sum_{n=1,3,5\ldots}^{\infty} \frac{2v_s}{n\pi}\sin(n\omega t) \quad (6)$$

For $n = 1$, the rms voltage of the fundamental component is

$$V_{r1} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{2v_s}{\pi}\sin(\omega t)\right)^2 dt} \quad (7)$$

where the frequency ω in **rad/s** is

$$\omega = \frac{2\pi}{T} \quad (8)$$

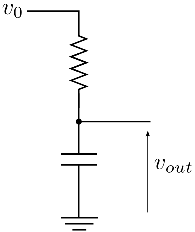
Fundamental harmonics

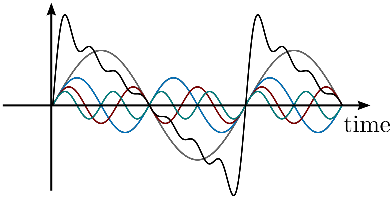
$$V_{r1}^2 = \frac{1}{T}\frac{4v_s^2}{\pi^2}\int_0^T \left(\frac{1}{2} - \frac{\cos(2\omega t)}{2}\right) dt \qquad (9)$$

After some straightforward manipulation

$$V_{r1} = \frac{2v_s}{\pi\sqrt{2}} = 0.45v_s \qquad (10)$$

v_0 may require filtering to eliminate higher harmonics





Half-bridge with RL load

For an LR load, the impedance is

$$Z = R + jn\omega L \tag{11}$$

The load current is

$$i_0 = \frac{v_0}{Z} \tag{12}$$

which gives

$$i_0 = \sum_{n=1,3,5..}^{\infty} \frac{2v_s}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n) \tag{13}$$

with $\theta_n = \tan^{-1}\left(\frac{n\omega L}{R}\right)$

The fundamental output power $(n = 1)$ is

$$P = (V_{r1})(I_{r1})\cos(\theta_n) = I_{r1}^2 R \tag{14}$$

Half-bridge with RL load

I_{r1} is the rms current of the first harmonic:

$$I_{r1} = \frac{2v_s}{\pi\sqrt{2}\sqrt{R^2 + (\omega L)^2}} \tag{15}$$

Thus the output power is

$$P_{r1} = \left[\frac{2v_s}{\pi\sqrt{2}\sqrt{R^2 + (\omega L)^2}}\right]^2 R \tag{16}$$

Higher harmonic ($n > 1$) are dissipated as heat.

Assuming a lossless inverter

$$\int_0^T v_s(t) i_s(t) dt = \int_0^T v_0(t) i_0(t) dt \tag{17}$$

Supply current

For high L and ω, i_0 is nearly sinusoidal and only the fundamental component provides power to the load:

$$\int_0^T v_s(t) i_s(t) dt = \int_0^T V_{r1} I_{r1} \cos(\theta_1) dt \tag{18}$$

v_s remains constant, thus the supply current is

$$I_s T = \frac{1}{v_s} \int_0^T V_{r1} I_{r1} \cos(\theta_1) dt \tag{19}$$

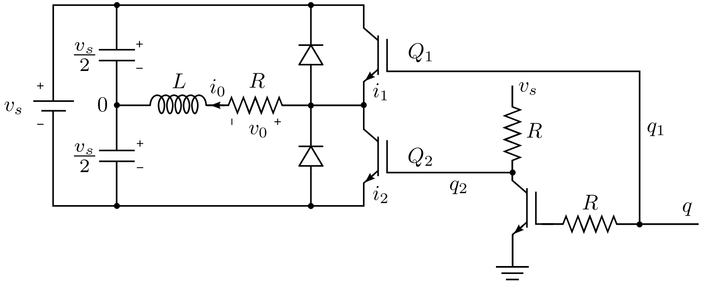
$$I_s = \frac{V_{r1}}{v_s} I_{r1} \cos(\theta_1) \tag{20}$$

$\rightarrow$ V_{01} is the fundamental rms output voltage

$\rightarrow$ I_{01} is the fundamental rms load current

$\rightarrow$ θ_1 is the load angle for $n = 1$

Gating sequence

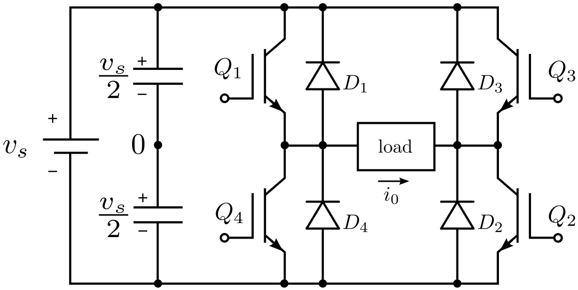


→ Generate a square-wave gating signal q_1

→ q_2 should be the logic inverter of q_1

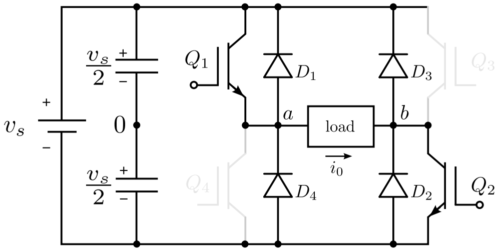
What is the function of the diodes?

Full bridge inverters



Full bridge inverters

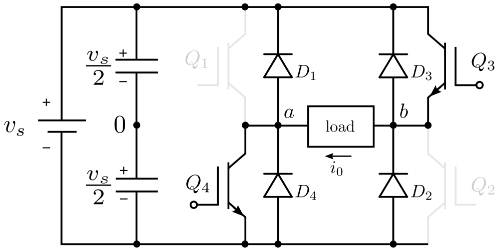
State 1 - Q_1 and Q_2 are on, Q_3 and Q_4 are off



v_{a0}	v_{b0}	v_0	Components conducting
			Q_1, Q_2 if $i_0 > 0$ D_1, D_2 if $i_0 < 0$

Full bridge inverters

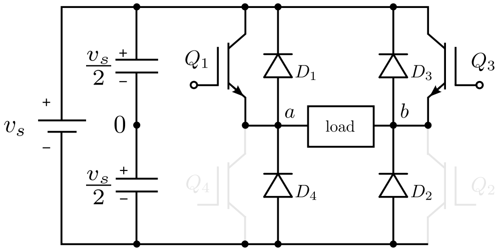
State 2 - Q_3 and Q_4 are on, Q_1 and Q_2 are off



v_{a0}	v_{b0}	v_0	Components conducting
			D_4, D_3 if $i_0 > 0$ Q_1, Q_3 if $i_0 < 0$

Full bridge inverters

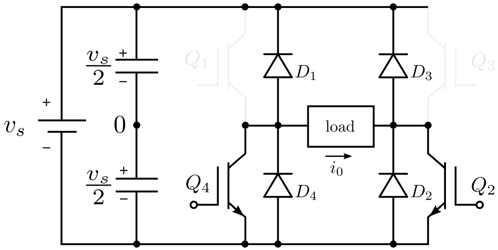
State 3 - Q_1 and Q_3 are on, Q_2 and Q_4 are off



v_{a0}	v_{b0}	v_0	Components conducting
			S_1, D_3 if $i_0 > 0$ D_1, Q_3 if $i_0 < 0$

Full bridge inverters

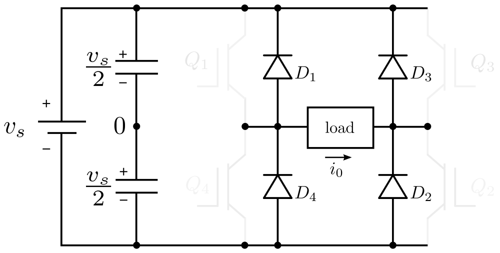
State 4 - Q_2 and Q_4 are on, Q_1 and Q_3 are off



v_{a0}	v_{b0}	v_0	Components conducting
			D_4, Q_2 if $i_0 > 0$ Q_4, D_2 if $i_0 < 0$

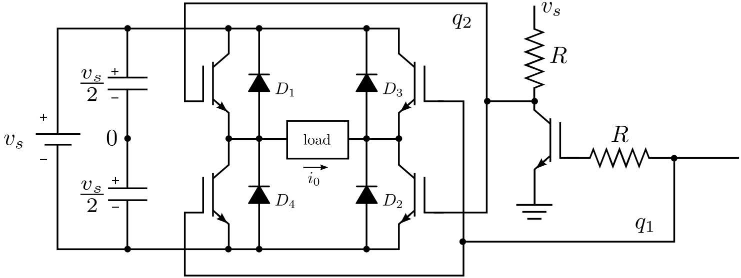
Full bridge inverters

State 5 - All switches are off



v_{a0}	v_{b0}	v_0	Components conducting
			D_4, D_3 if $i_0 > 0$ D_1, D_2 if $i_0 < 0$

Gating sequence



→ Generate a square-wave gating signal q_1

→ q_2 should be the logic inverter of q_1

Full bridge inverter

The rms output voltage is

$$v_0 = \sqrt{\frac{1}{T}\int_0^T v_s^2 dt} = v_s \tag{21}$$

Following the same procedure as for the half-bridge:

$$v_0 = \sum_{n=1,3,5\ldots}^{\infty} \frac{4v_s}{n\pi}\sin(n\omega t) \tag{22}$$

The fundamental component is

$$V_{1r} = \frac{4v_s}{\pi\sqrt{2}} = 0.9v_s \tag{23}$$

For an *RL* load, the **instantaneous** current is

$$i_0 = \sum_{n=1,3,5\ldots}^{\infty} \frac{4v_s}{n\pi\sqrt{R^2 + (n\omega L)^2}}\sin(n\omega t - \theta_n) \tag{24}$$

with $\theta_n = \tan^{-1}\left(\frac{n\omega M}{L}\right)$

Measures of performance

Harmonic factor: The individual harmonic contribution

$$HF_n = \frac{V_{rn}}{V_{r1}} \tag{25}$$

V_{rn} is the n^{th} rms harmonic component

Harmonic distortion: Closeness between a waveform and its fundamental component

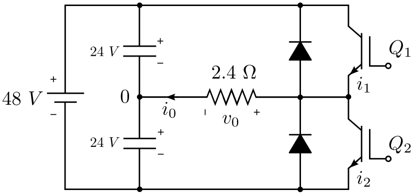
$$THD = \frac{1}{v_{1r}}\sqrt{\sum_{n=3,5,7\ldots}^{\infty} V_{on}^2} \tag{26}$$

Distortion factor: Harmonics remaining after filtering

$$DF = \frac{1}{v_{1r}}\sqrt{\sum_{n=3,5,7\ldots}^{\infty} \left(\frac{V_{on}}{n^2}\right)^2} \tag{27}$$

Exercise 20

Consider the following single phase half bridge inverter with a resistive load of $R = 2.4\ \Omega$



Determine:

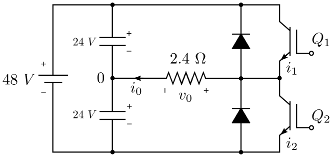
(a) The rms output voltage at the fundamental frequency

(b) The output power

(c) The average and peak current of each transistor for a 50% duty cycle

(d) The average supply current I_s

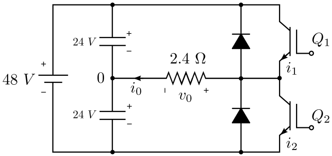
Exercise 20 - continued



(a) The rms output voltage at the fundamental frequency

(b) The output power

Exercise 20 - continued

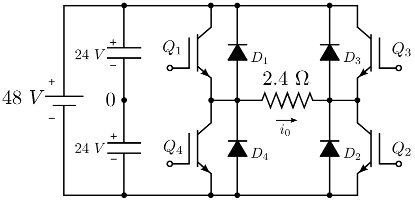


(c) The average and peak current of each transistor for a 50% duty cycle

(d) The average supply current I_s

Exercise 21

Consider the following single phase full bridge inverter with a resistive load of $R = 2.4\ \Omega$



Determine:

(a) The rms output voltage at the fundamental frequency

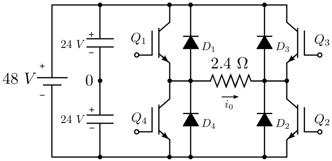
(b) The output power

(c) The average and peak current of each transistor for a 50% duty cycle

(d) The average supply current I_s

(e) The harmonic distortion and harmonic factor

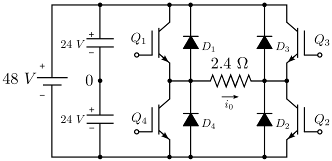
Exercise 21 - continued



(a) The rms output voltage at the fundamental frequency

(b) The output power

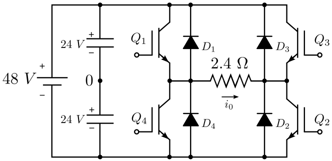
Exercise 21 - continued



(c) The average and peak current of each transistor for a 50% duty cycle

(d) The average supply current I_s

Exercise 21 - continued

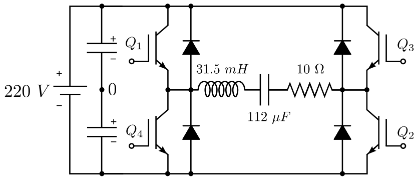


(e) The harmonic distortion and harmonic factor

Exercise 21 - continued

Exercise 22

Consider the following single phase full bridge inverter with a LRC load. The switching frequency is 60 Hz.



(a) Express the instantaneous load current as a Fourier series

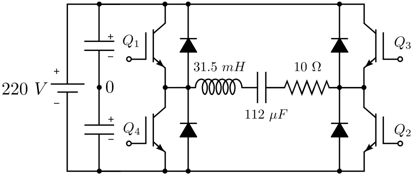
(b) Calculate the rms load current at the fundamental frequency

(c) Calculate the peak and rms load current

(d) The load power and the fundamental output power

(e) The average supply current

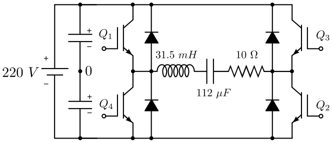
Exercise 22 - continued



(a) Express the instantaneous load current as a Fourier series

Exercise 22 - continued

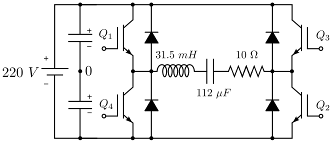
Exercise 22 - continued



(b) Calculate the rms load current at the fundamental frequency

(c) Calculate the peak and rms load current

Exercise 22 - continued

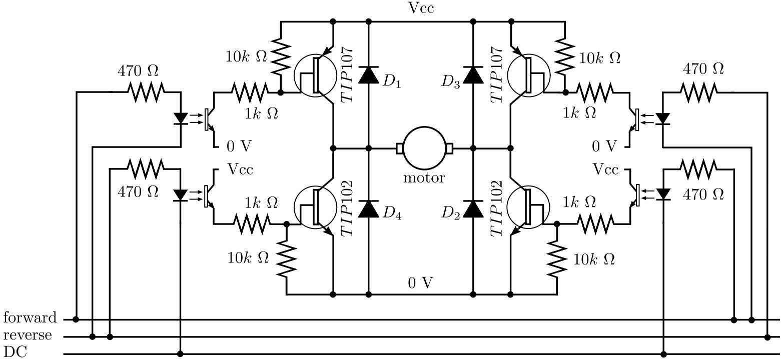


(d) The load power and the fundamental output power

(e) The average supply current

Exercise 23

The full bridge inverter is used to control a DC motor using the configuration shown. Each PNP transistor is activated using an opto-isolator - an electronic component that transfers electrical signals between two isolated circuits by using light.



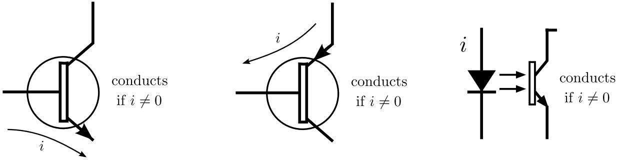
Describe the working of the inverter as a function of the "forward", "reverse", and duty-cycle "DC" signals.

Exercise 23 - continued

Datasheets

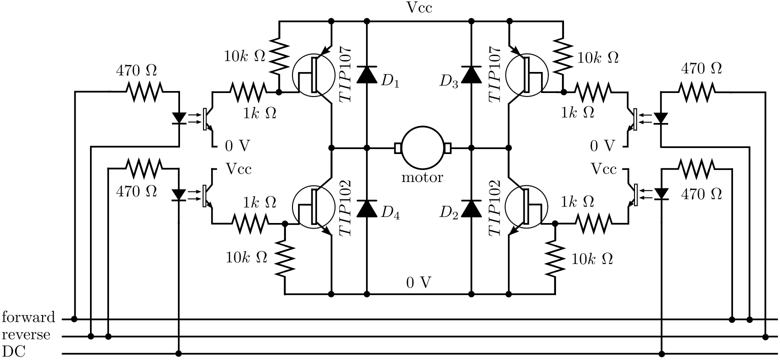
Opto-couplers: `https://goo.gl/BQV5B9`

Transistors: `https://goo.gl/6SBMmZ`



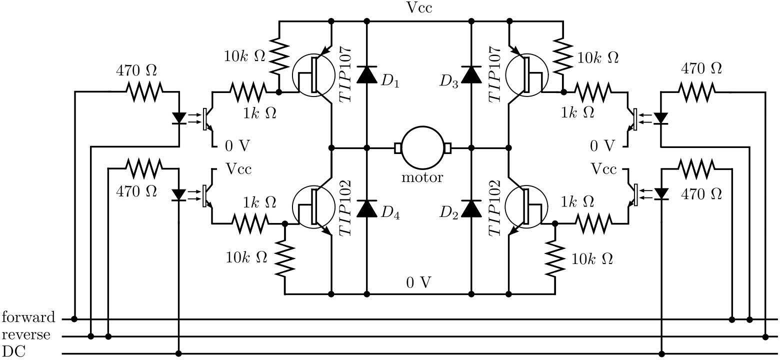
$TIP102$ $TIP107$ $PS2501$

Exercise 23 - continued



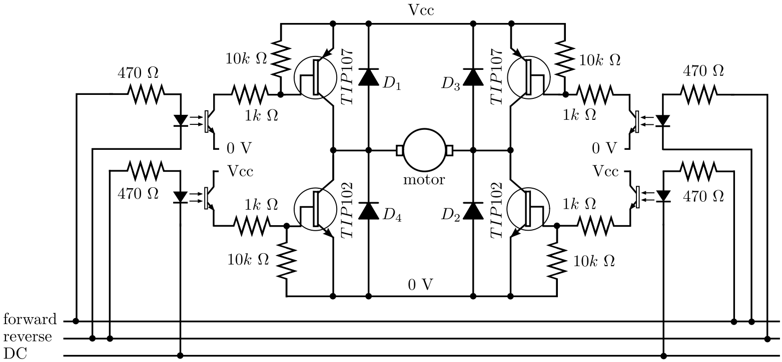
Description	
Forward	1
Reverse	0
DC	

Exercise 23 - continued



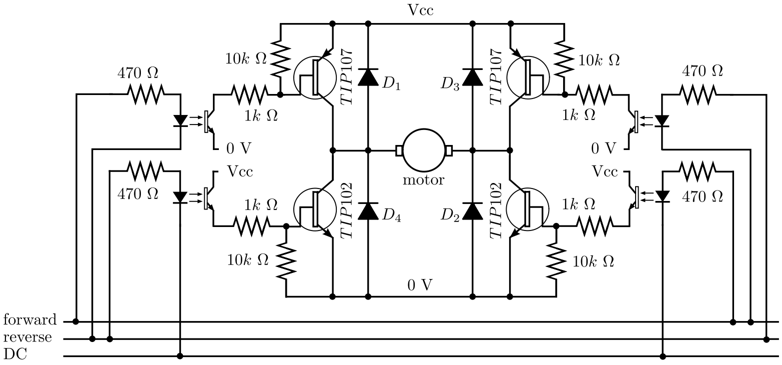
Description	
Forward	0
Reverse	1
DC	

Exercise 23 - continued



Description	
Forward	0
Reverse	0
DC	

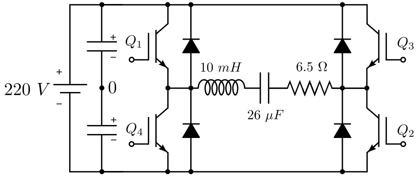
Exercise 23 - continued



Description	
Forward	1
Reverse	1
DC	

Exercise 24

Consider the following single phase full bridge inverter with a LRC load. The switching frequency is 400 Hz.



(a) Express the instantaneous load current as a Fourier series

(b) Calculate the rms load current at the fundamental frequency

(c) Calculate the peak and rms load current

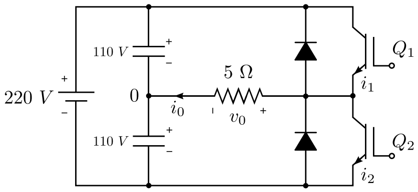
(d) The load power and the fundamental output power

(e) The average supply current

Exercise 24 - continued

Exercise 25

Consider the following single phase half bridge inverter with a resistive load of $R = 5\ \Omega$



Determine:

(a) The rms output voltage at the fundamental frequency

(b) The output power

(c) The average and peak current of each transistor for a 50% duty cycle

(d) The average supply current I_s

(e) The harmonic distortion and harmonic factor

Exercise 25 - continued

Next class...

- Pulse width modulation

Additional supporting materials for Lecture 6:

DC/AC converters: `https://goo.gl/poscjj`

H-bridge motor control: `https://goo.gl/VVFmDc`

H-bridge circuit analysis: `https://goo.gl/gGTKzb`