METE 3100U
Actuators and Power Electronics

Lecture 3

Transformers

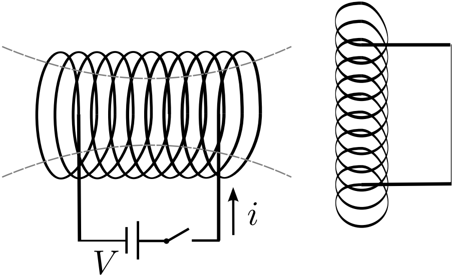
Outline of Lecture 3

In today's lecture we will

- Review the principles of transformers
- Model and analyse a transformer

Faraday's law - from lecture 1

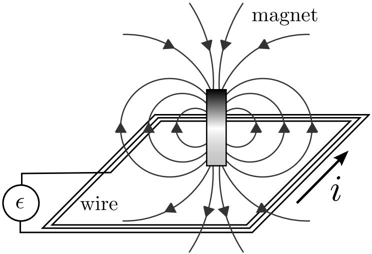
A change in the magnetic environment of a coil will cause a voltage (emf) to be induced in the coil



Electromotive force (EMF): drives a current in electrical circuits

Lenz's law - from lecture 2

An induced electromotive force ϵ gives rise to a current i whose magnetic field opposes the change in original magnetic flux.



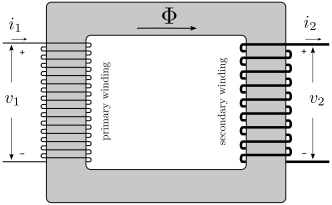
Lenz's law is shown with the negative sign in Faraday's law of induction

$$\epsilon = -N\frac{\partial \Phi}{\partial t} \quad (1)$$

Ohm's law yields:

$$\epsilon = iR \quad (2)$$

Ideal transformer



$$v_1 = \epsilon_1 = N_1 \frac{d\Phi}{dt}, \qquad v_2 = \epsilon_2 = N_2 \frac{d\Phi}{dt} \tag{3}$$

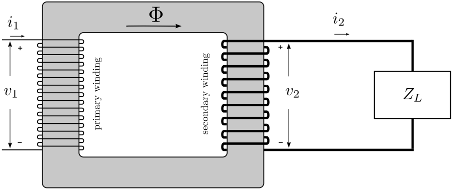
$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a \tag{4}$$

→ Windings have zero resistance and inductance

→ No flux leakage

→ The permeability of the core is infinite (zero reluctance)

Ideal transformer



When a load Z_L is attached, a current i_2 flow in the secondary coil.

$$\sum \epsilon = 0 \rightarrow N_1 i_1 - N_2 i_2 = 0 \quad (5)$$

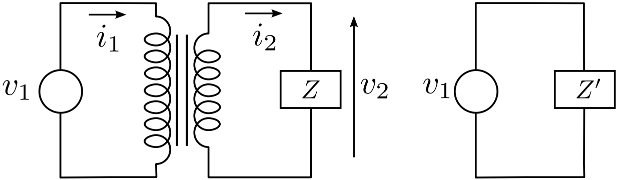
$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a} \quad (6)$$

The instantaneous power is conserved:

$$v_1 i_1 = v_2 i_2 \quad (7)$$

v is the root mean square of a sinusoidal voltage.

Impedance transfer



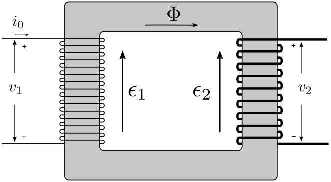
$$Z_2 = \frac{V_2}{i_2}, \qquad Z_1 = \frac{V_1}{i_1} \tag{8}$$

Since $V_1 = V_2 a$, and $i_1 = i_2/a$, the input impedance becomes

$$Z_1 = \frac{V_2 a}{i_a/a} = a^2 \frac{V_2}{i_2}$$

$$Z' =$$

Practical transformer



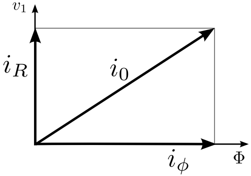
The no-load current i_0:

$\rightarrow$ Magnetizes the transformer's core (i_ϕ)

$\rightarrow$ Compensates the magnetic losses such as eddy current (i_R)

$$\Phi(t) = \Phi_{max} \sin(\omega t)$$

$$v_1(t) = N\frac{d\Phi}{dt} = N\Phi_{max}\omega \cos(\omega t)$$



Practical transformer

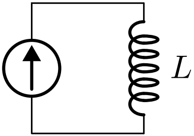
The inductance L of the windings is defined as the flux linkage λ of the coil per Ampère of its current.

Flux linkage: N times the magnetic flux:

$$\lambda = N\Phi \tag{9}$$

From Lenz' law, the inductance is a property of a conductor which opposes any change in current through the conductor:

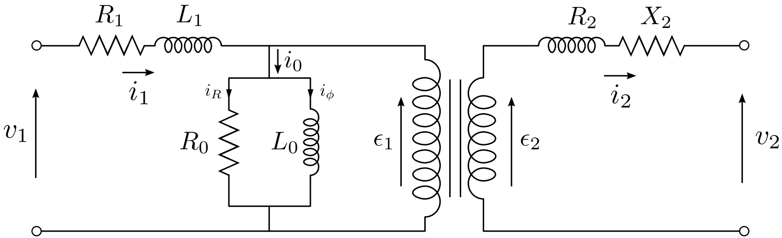
$$L = \frac{\lambda}{i} = \frac{N\Phi}{i} \tag{10}$$



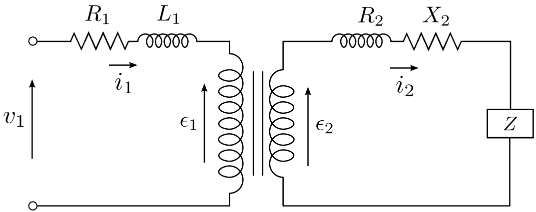
$$L = \frac{NBA}{i} = \frac{N\mu HA}{i} = \frac{NBA}{\frac{H\ell}{N}} = \frac{N^2}{\frac{\ell}{\mu A}} \tag{11}$$

$$L = \frac{N^2}{\Re} \tag{12}$$

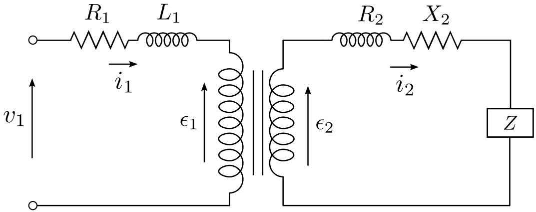
Equivalent circuit



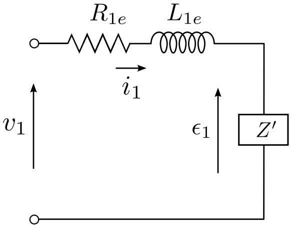
If the transformer is loaded, then $i_1 >> i_0$:



Equivalent circuit



Secondary impedance refereed to the primary

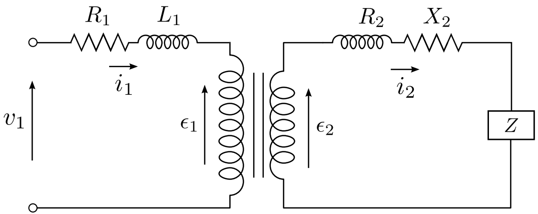


$$R_{1e} = R_1 + R_2\left(\frac{N_1}{N_2}\right)^2$$

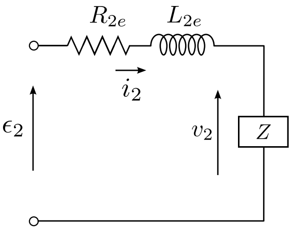
$$L_{1e} = L_1 + L_2\left(\frac{N_1}{N_2}\right)^2$$

$$Z' = Z\left(\frac{N_1}{N_2}\right)^2$$

Equivalent circuit



Primary impedance refereed to the secondary



$$R_{2e} = R_2 + R_1 \left(\frac{N_2}{N_1}\right)^2$$

$$L_{2e} = L_2 + L_1 \left(\frac{N_2}{N_1}\right)^2$$

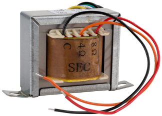
Transformer rating

→ Kilovolt-Ampère (kVa): 10 kVA, 1100/110

→ Primary: $v_1 = 1100$ V,

→ Secondary: $v_2 = 110$ V,

→ Turn ratio: $a = 1100/110 = 10$



Input current:

Output current:

Note that in practice $i_0 \neq 0$

Exercise 04

Circuit tests performed on a 10 kVA, 2200/220 V, 60 Hz transformer as follows:

→ Test 1 - No load test: The terminals of the high voltage side are open

→ Test 2 - Short circuit test: The terminals of the low voltage side are shorted

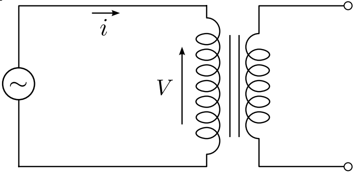
The obtained results are:

	Test 1	Test 2
Applied voltage	220 V	150 V
Input current	2.5 A	4.55 A
Dissipated power	100 W	215 W

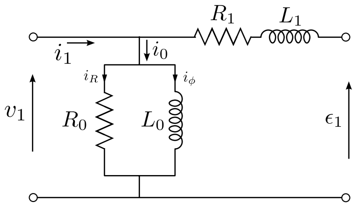
Determine the parameters for the approximate equivalent circuits referred to the low-voltage side and the high-voltage side.

Exercise 04 - continued

Step 1: No-load test



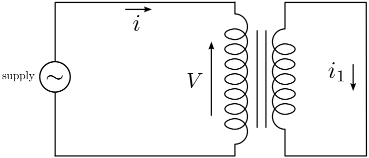
no-load circuit test



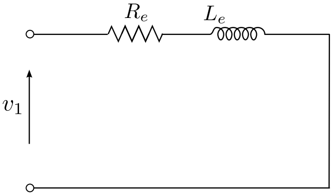
no-load equivalent circuit

Exercise 04 - continued

Step 2: Short circuit test



short circuit test

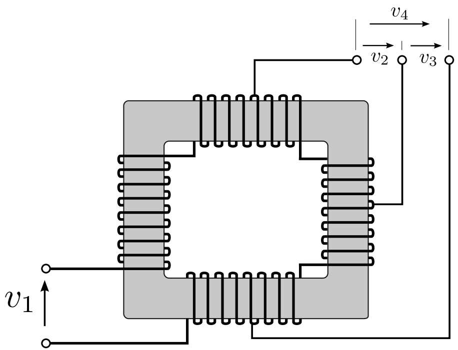


equivalent shorted circuit

Autotransformers

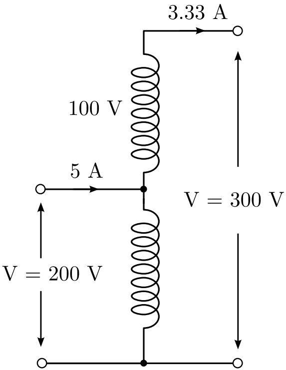
→ Allows for a variable AC voltage at the secondary.

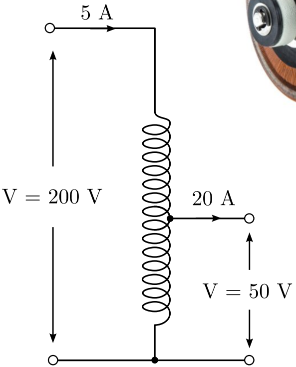
→ The primary and secondary are connected.



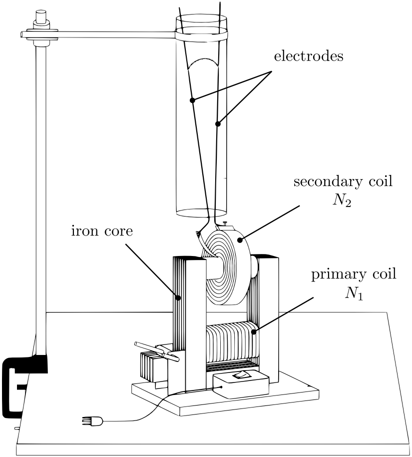
Note that $v_2 \approx v_3 < v_4 < v_1$

Autotransformers



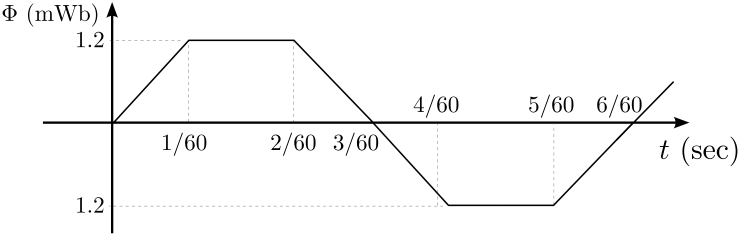


Experiment - Jacob's ladder

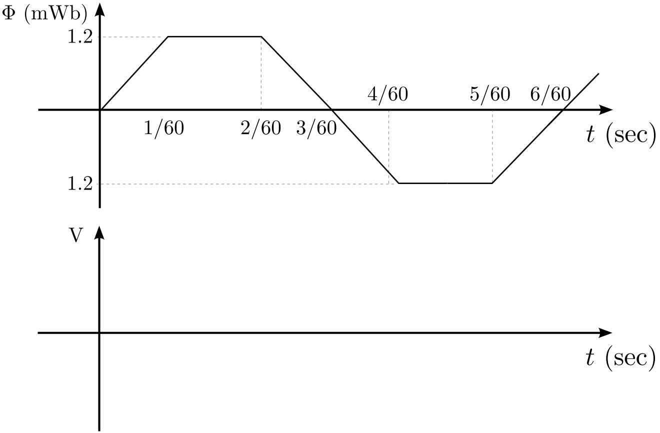


Exercise 05

The flux in a single phase transformer varies with time as shown in the graph. The primary coil has 400 turns and the secondary coil has 100 turns. Sketch the waveform of the induced voltage e_1 in the primary winding.

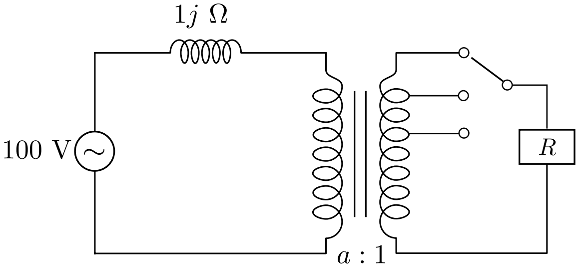


Exercise 05 - continued



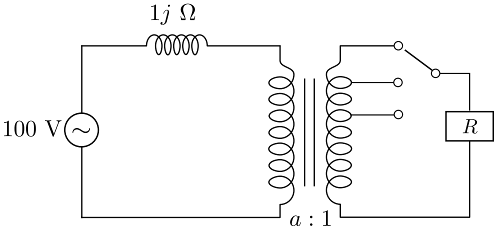
Exercise 06

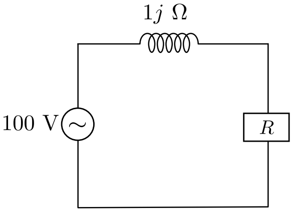
A resistive load varies from 1 to 0.5 Ω. The generator can be modelled as an ideal transformer with a constant voltage of 100 V in series with an inductive reactance. For maximum power transfer to the load, the effective load resistance seen at the transformer primary must equal the series impedance of the generator, that is, the referred value of R to the primary side is always 1 Ω.



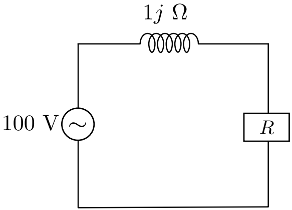
Determine the turn ratio and the load voltage for maximum power transfer.

Exercise 06 - continued



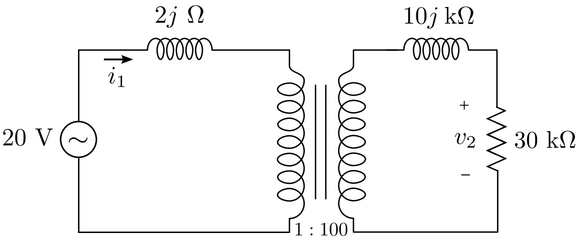


Exercise 06 - continued

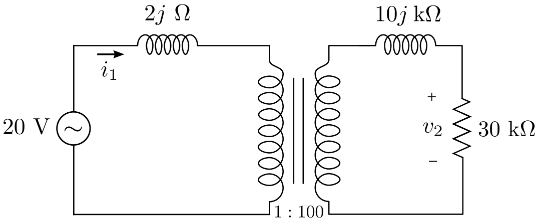


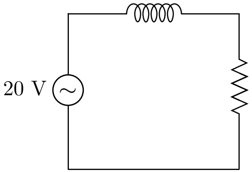
Exercise 07

For the circuit shown, consider the transformer to be ideal with a turn ratio of 1:100. Calculate the actual load voltage v_2 and the supply current i_1.

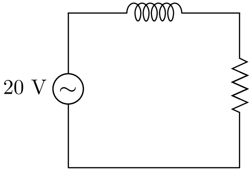


Exercise 07 - continued



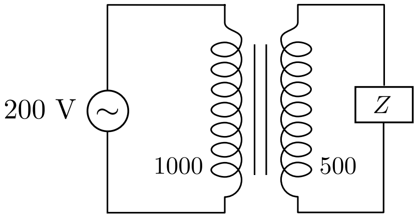


Exercise 07 - continued



Exercise 08

The transformer has 1000 turns on the primary and 500 turns on the secondary. The primary winding is connected to a 200 V supply and the secondary winding is connected to a 5 kVA load. The transformer can be considered as ideal.

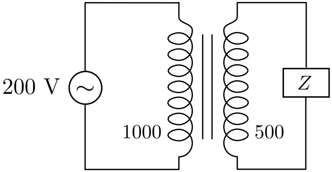


(a) Determine the load voltage

(b) Determine the load impedance

(c) Determine the load impedance referred to the primary

Exercise 08 - continued



Exercise 09

A single phase transformer has 500 turns in the primary winding. When it is connected to a 120 V, 60 Hz power supply, the no-load current is 1.6 A and the no-load power is 80 W. Neglect the winding resistance and leakage reactance of the winding. Calculate

(a) The core loss current i_c

(b) The magnetizing current i_m

(c) The peak value of the cure flux Φ_m

Exercise 09 - continued

Next class...

- Diode rectifiers (AC/DC converter)

Additional supporting materials for Lecture 3:

Transformer loading: `https://goo.gl/kRnViR`

Autotransformer inner workings: `https://goo.gl/zCQf9j`

Three-phase transformer: `https://goo.gl/8Ddw2C`