METE 3100
Actuators and Power Electronics

Lecture 2

Magnetic Circuits

Lecture outline

By the end of today's lecture you should be able to

→ Understand the relation between current and magnetic field

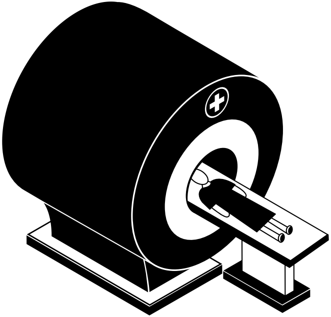
→ Understand the concept of electromagnetic induction

→ Model a simple electromagnetic circuit

Applications

Magnetic resonance imaging is a type of scan that uses strong magnetic fields and radio waves to produce detailed images of the inside of the body.

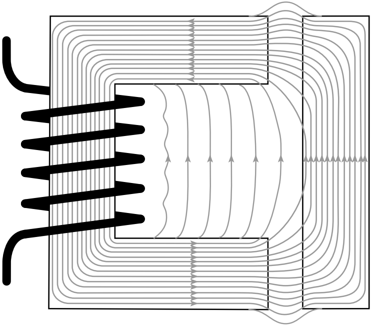
How is the magnetic field generated and oriented ?



Applications

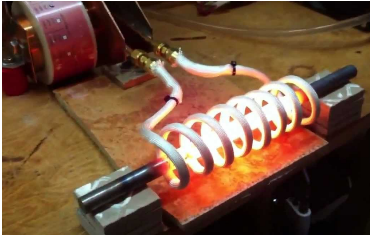
An electromagnet is a type of magnet in which the magnetic field is produced by an electric current.

How can we model and design an electromagnet ?



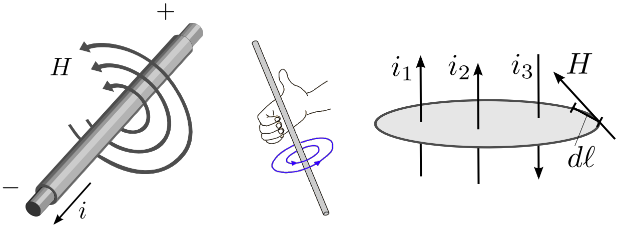
Applications

Induction heating is the process of heating an electrically conducting object by electromagnetic induction. What is the relation between current and heat?



Ampère's circuit law

When a conductor carries a current i, a magnetic field H is produced around it.

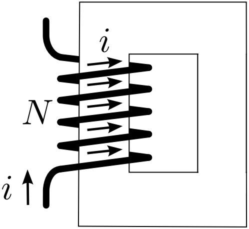


The integral of the field around a closed path gives the current

$$\oint H \cdot dl = \oint |H| d\ell \cos\theta = \sum i = i_1 + i_2 - i_3 = \sum i \tag{1}$$

Ampère's circuit law

Consider the closed-loop magnetic circuit having a coil with N turns.



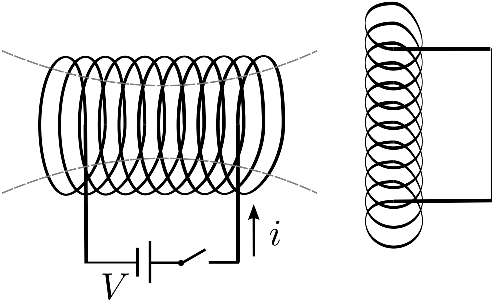
From Ampère's law and by neglecting flux leakages:

$$\mathcal{F} = \oint H \cdot dl = H\ell = \tag{2}$$

$\mathcal{F}$ is called the magnetomotive force (mmf).

Faraday's law

A change in the magnetic environment of a coil will cause a voltage (emf) to be induced in the coil

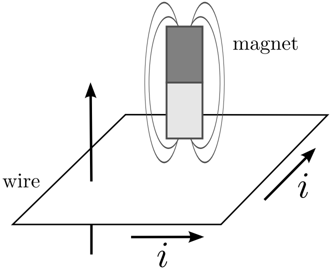


Electromotive force (EMF): drives a current in electrical circuits

Magnetomotive force (MMF) drives magnetic flux through magnetic circuits.

Lenz's law

An induced electromotive force ϵ gives rise to a current i whose magnetic field opposes the change in original magnetic flux.



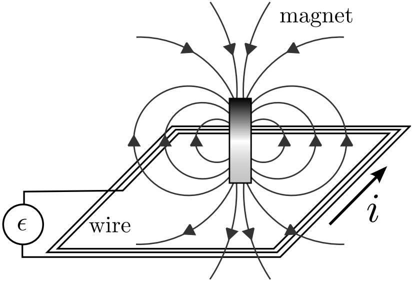
Lenz's law is shown with the negative sign in Faraday's law of induction

$$\epsilon = -\frac{\partial \Phi}{\partial t} \tag{3}$$

Ohm's law yields:

$$\epsilon = iR \tag{4}$$

Electromotive force



In the case of a solenoid:

$$\epsilon = -N\frac{d\Phi}{dt} \tag{5}$$

$$i = \frac{\epsilon}{R} \tag{6}$$

R is the resistance of the solenoid wire.

Electromotive force

Magnetic flux density

The magnetic field H produces a magnetic flux density B

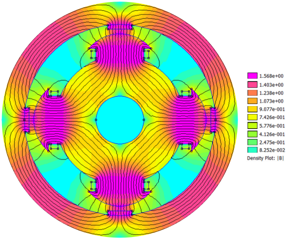
B: the number of magnetic lines of flux that pass through a surface.

$$B = \mu H = \mu_r \mu_0 H \tag{7}$$

- $\rightarrow \mu$: permeability of the medium
- $\rightarrow \mu_0 = 4\pi \times 10^{-7}$ M/m: permeability of the vacuum
- $\rightarrow \mu_r$ **relative** permeability of the medium

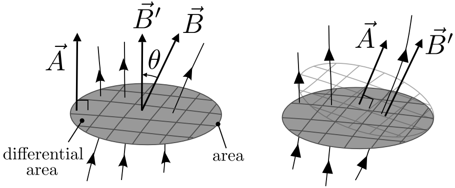
Unit:

- $\Rightarrow$ T (tesla) or $\mathrm{Wb/m^2}$
- $\Rightarrow$ G (gauss): 1 T = 10,000 gauss.



Magnetic flux

The magnetic flux $\vec{\Phi}$ through a surface $\vec{A}$ is the surface integral of the magnetic field $\vec{B}$ passing through that surface.



$$\Phi = \int \vec{B} \cdot d\vec{A} \tag{8}$$

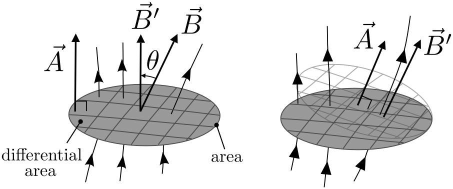
The electromotive force is

$$\epsilon = -\frac{\partial \vec{\Phi}}{\partial t} \tag{9}$$

Thus

$$\epsilon = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{A} \tag{10}$$

Magnetic flux



For a closed surface and constant magnetic field:

$$\Phi = \int \vec{B} \cdot d\vec{A} = BA \ \ [\text{Volt.sec} = \text{Weber}] \tag{11}$$

Since $B = \mu H$ and $Ni = H\ell$, the magnetic flux can be rewritten as

$$\Phi = (\mu H)A = \mu \left(\frac{Ni}{\ell}\right) A = \frac{Ni}{\frac{\ell}{\mu A}} \tag{12}$$

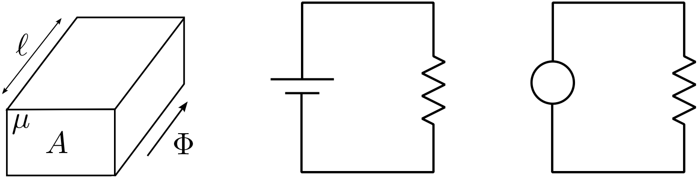
The term $\frac{\ell}{\mu A} = \Re$ is called the magnetic **reluctance**.

Reluctance of a magnetic path

Magnetic circuits are analogue to electric circuits

The magnetic reluctance is a scalar extensive quantity, akin to electrical resistance

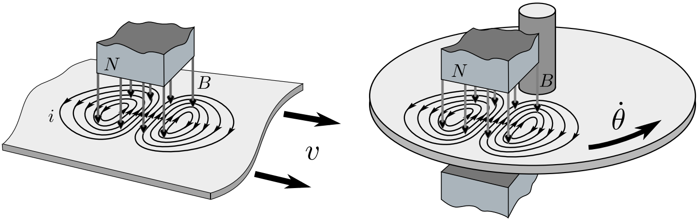
$$\Re = \frac{\ell}{\mu A} \tag{13}$$



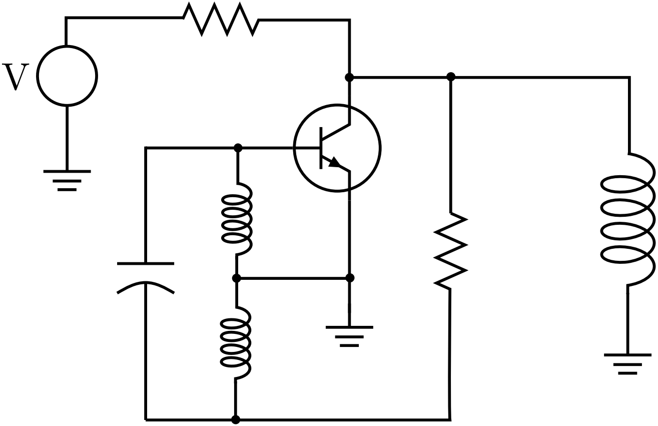
$$\Phi = \frac{Ni}{\Re} \tag{14}$$

Foucault currents - or Eddy current

A localized electric current induced in a conductor by a varying magnetic field.



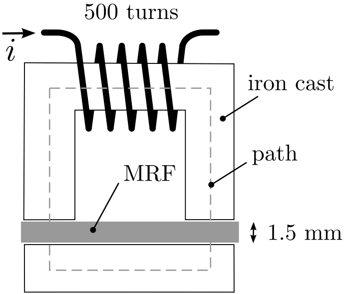
Experiment



http://www.falstad.com/circuit/e-hartley.html

Exercise 01

A magnetorheological fluid (MRF) is a type of smart material in a carrier fluid. When subjected to a magnetic field, the fluid greatly increases its apparent viscosity, to the point of becoming a viscoelastic solid[1]. The required magnetic field density to achieve solid state is 0.8 T. If the length of the mean core path shown is 360 mm, and the relative permeability of the fluid and core are $\mu_r = 4$ and $\mu_r = 1178$, calculate the required current in the coil.



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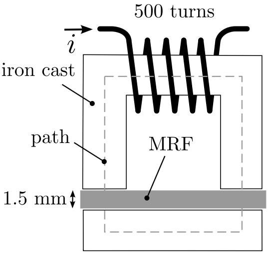
[1] `http://www.biomechatronics.ca/research/#brakes`

Exercise 01 - continued

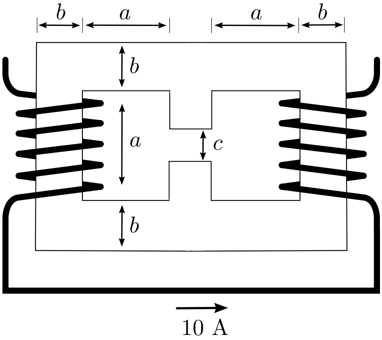
$B_{mrf} = 0.8$ T, $\ell_{iron} = 0.36$ m,

Determine i



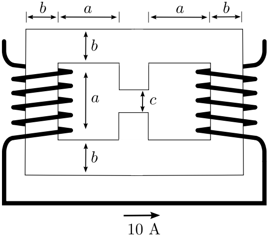
Exercise 02

In the magnetic circuit shown, the μ_r of the ferromagnetic path is 1200. The magnetic path has a square cross-sectional. If $a = 0.5$, $b = 0.02$, and $c = 0.05$ m, and each winding has 500 turns, determine the air gap flux density.

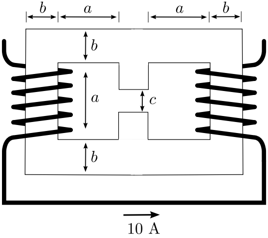


Exercise 02 - continued

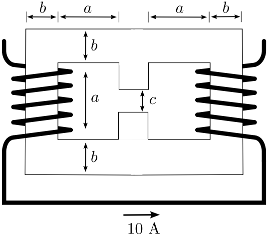
$\mu_r = 1200$, $a = 0.5$, $b = 0.02$, and $c = 0.05$ m, and $N = 500$ turns,



Exercise 02 - continued

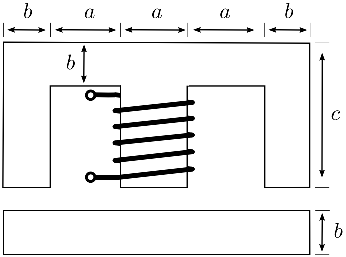


Exercise 02 - continued



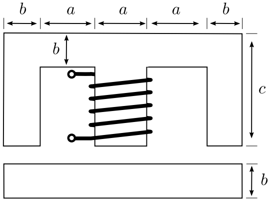
Exercise 03

The electromagnet shown can be used to lift a length of steel strip. The coil has 500 turns and can carry a current of 20 Amps. The magnetic material has negligible reluctance at flux densities up to 1.4 T. Determine the maximum air gap for which a flux density of 1.4 T can be established.



Take: $a = 20, b = 10, c = 50$ mm.

Exercise 03 - continued



Next episode...

- Transformers