

METE 3100U  
Actuators and Power Electronics

Lecture 17  
Induction Machines 2/2

## Outline of Lecture 17

By the end of today's lecture, you should be able to

- Model an induction machine
- Find an equivalent transformer model
- Estimate the torque of an induction machine

## Applications

The Birmingham Maglev was opened in 1984 and was the first commercial Maglev transport system in the world.



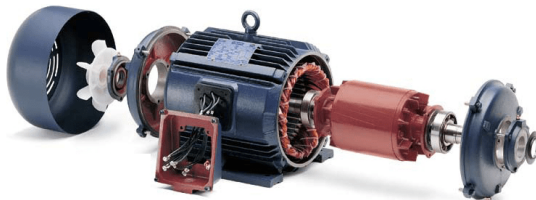
## Applications

AirTrain JFK trains propel themselves using an aluminium induction strip placed between the rails.



# Applications

What is the torque developed by an induction machine?



## Operating principle - review

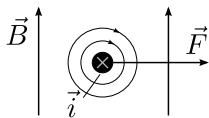
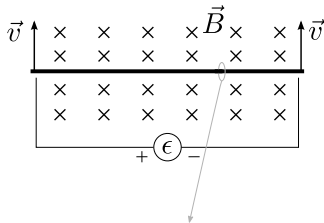
**Faraday's law:** A voltage is induced in the moving conductor:

$$\epsilon = Blv$$

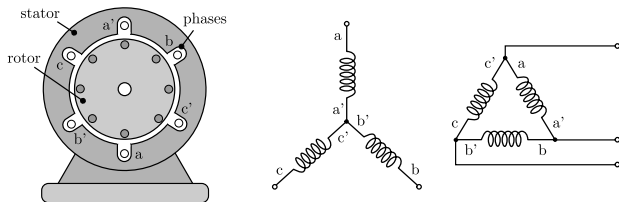
The interaction between the magnetic field  $B$  and the current generates a force

$$F = Bli$$

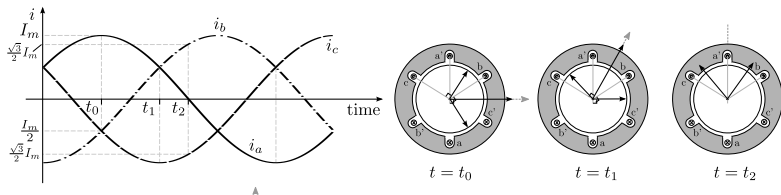
A torque is produced as a result of the interaction of those two magnetic fields



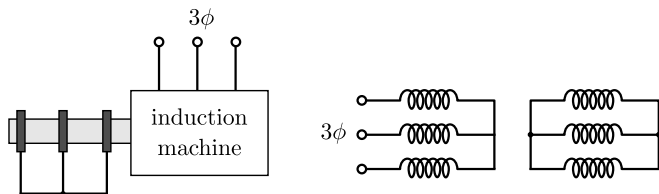
## Operating principle - review



The current in each coil produces a rotating mmf wave



## Equivalent circuit model



Stator - per phase

Rotor - per phase

$V_1$  terminal voltage

$R_1$  winding resistance

$L_1$  winding inductance

$E_1$  induced mmf

$L_m$  magnetizing inductance

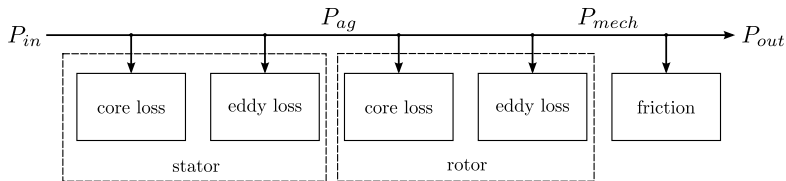
$R_c$  core loss resistance

$E_2$  induced mmf ( $s=1$ )

$L_2$  leakage inductance

$R_2$  circuit resistance

## Power flow



Stator - per phase

Rotor - per phase

$V_1$  terminal voltage

$R_1$  winding resistance

$L_1$  winding inductance

$E_1$  induced mmf

$L_m$  magnetizing inductance

$R_c$  core loss resistance

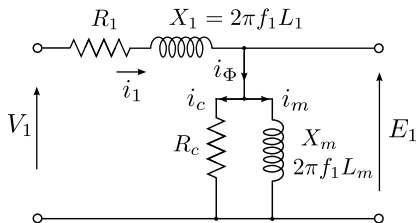
$E_2$  induced mmf ( $s=1$ )

$L_2$  leakage inductance

$R_2$  circuit resistance

## Equivalent circuit model - stator

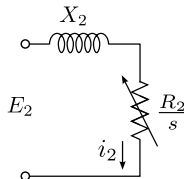
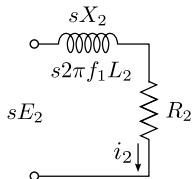
The stator equivalent circuit can be represented as a transformer



- $i_\Phi$  is larger than in a transformer. Why?
- $X_1$  is high due to the air gap
- $E_1$  is the back emf induced by the rotor
- $f_1$  is the frequency of the terminal voltage

## Equivalent circuit model - rotor

The rotor can be presented as a RL circuit:



→ Note that the rotor frequency is  $f_2 = sf_1$

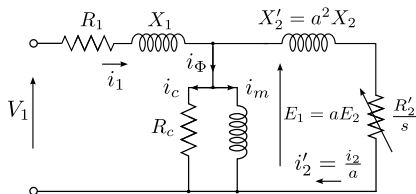
→ The rotor emf is  $E_2 = sE_{2s}$  ( $E_{2s}$  is at standstill)

From Kirchhoff's law the rotor current is

$$i_2 = \frac{sE_2}{R_2 + jsX_2} = \frac{E_2}{\frac{R_2}{s} + jX_2} \quad (1)$$

## Equivalent circuit model - rotor

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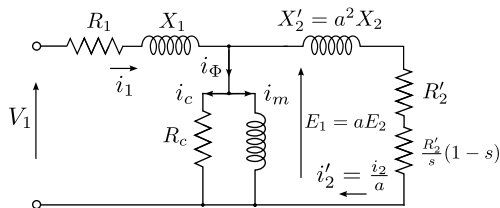
$$i_2 = \frac{sE_2}{R_2 + jsX_2} = \frac{E_2}{\frac{R_2}{s} + jX_2} \quad (2)$$

The power in the rotor is

$$P = i_2^2 \frac{R_2}{s} = \frac{P_2}{s} \quad (3)$$



## Equivalent circuit model



The slip-dependent resistance is the mechanical power per phase

$$\begin{aligned}
 P_m &= i_2^2 \frac{R_2}{s} (1-s) \\
 &= (1-s)P \\
 &= \frac{1-s}{s} P_2
 \end{aligned}$$

→  $P$ : power that crosses the air gap (per phase)

→  $P_2$ : power at standstill (per phase)

## Mechanical power

The total mechanical power is

$$\begin{aligned}P_t &= 3P_m = 3i_2^2 \frac{R_2}{s}(1-s) \\ &= 3i_1^2 \frac{R_2'}{s}(1-s)\end{aligned}$$

Given the motor torque  $T$  and angular velocity  $\omega$ :

$$P_t = T\omega \rightarrow T = \frac{P_t}{\omega} \quad (7)$$

The angular velocity is

$$\omega = \omega_s(1-s) = \frac{2\pi}{60} n = \frac{2\pi}{60} n_s(1-s) \quad (8)$$

Thus the torque per phase is

$$T = \frac{1}{\omega} \left[ i_2^2 \frac{R_2}{s}(1-s) \right] = \frac{1}{\omega_s} \left[ i_2^2 \frac{R_2}{s} \right] \quad (9)$$

## Mechanical power

The torque per phase is

$$T = \frac{1}{\omega_s} \left[ i_2^2 \frac{R_2}{s} \right]$$

Using the equivalent circuit, the current in the rotor is

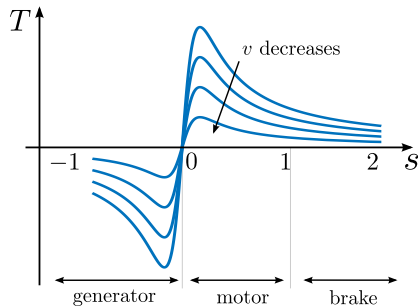
$$|i_2| = \frac{V_1}{\sqrt{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2}} \quad (10)$$

which gives the torque per phase as

$$T = \frac{1}{\omega_s} \frac{V_1^2}{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2} \frac{R_2'}{s} \quad (11)$$

## Torque per phase

$$T = \frac{1}{\omega_s} \frac{V_1^2}{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2} \frac{R_2'}{s}$$

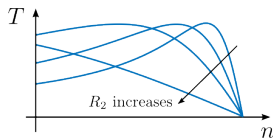


The total torque developed by the machine is  $3T$ .

## Maximum torque

Neglecting winding losses, the torque becomes

$$T \approx \frac{1}{\omega_s} \frac{V_1^2}{(X_1 + X_2')^2} \frac{R_2'}{s}$$



The slip  $s_m$  that provides maximum torque can be found as

$$\frac{\partial T}{\partial s} = 0 \rightarrow s_m = \frac{R_2'}{\sqrt{R_1^2 + (X_1 + X_2')^2}} \quad (12)$$

And the maximum torque at  $s = s_m$  is

$$T_m = \frac{1}{2\omega_s} \frac{V_1^2}{R_1 + \sqrt{R_1^2 + (X_1 + X_2')^2}} \quad (13)$$

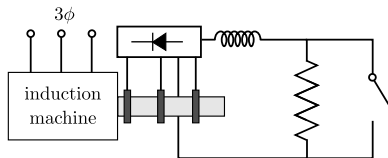
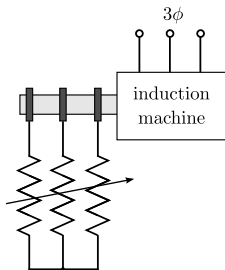
→ Maximum torque is independent of  $R_2$

→ From (12),  $R_2$  determines **the speed at which the maximum torque will occur**

## Effect of rotor resistance

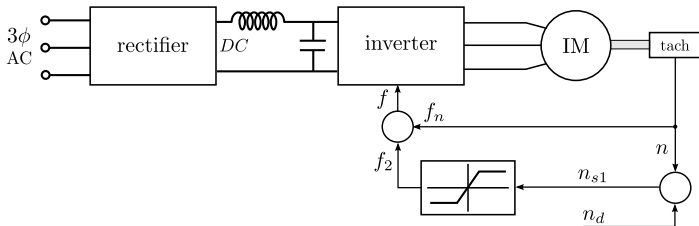
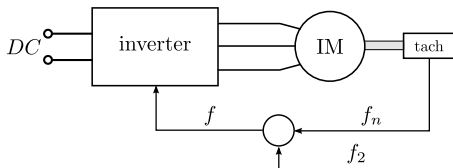
$$s_m = \frac{R'_2}{\sqrt{R_1^2 + (X_1 + X'_2)^2}},$$

$$T_m = \frac{1}{2\omega_s} \frac{V_1^2}{R_1 + \sqrt{R_1^2 + (X_1 + X'_2)^2}}$$



$$R_e = R(1 - k) \tag{14}$$

## Closed-loop control



## Exercise 79

A 3-phase, 4 pole, 60 Hz, 11kW, 460V, 1728 rpm induction motor delivers rated output power to a load connected to its shaft. The windage and friction loss of the motor is 750 W.

Determine:

- (a) The mechanical power developed by the motor
- (b) The air gap power
- (c) The rotor copper loss

## Exercise 79 - continued

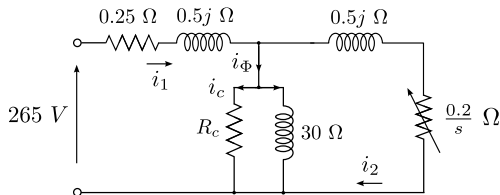
(a) The mechanical power developed by the motor

(b) The air gap power

(c) The rotor copper loss

## Exercise 80

A 3-phase, 460 V, 60 Hz, 4-pole wound-rotor induction machine has the parameters shown. The core losses due to  $R_c$  can be neglected (i.e.,  $i_c = 0$ ).

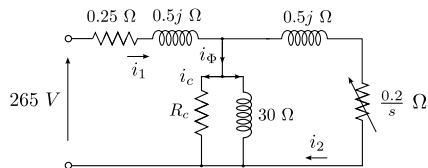


Calculate:

- The starting current
- The starting torque
- Full load slip and current if  $n = 1740$  rpm.
- Full load torque
- The resistance that must be connected in the rotor so that the maximum torque occurs at start.

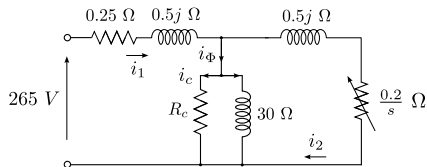
## Exercise 80 - continued

(a) The starting current



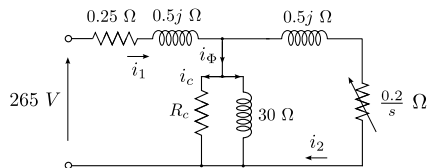
## Exercise 80 - continued

(b) The starting torque



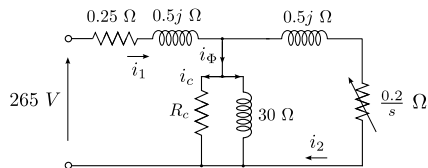
## Exercise 80 - continued

(c-1) Full load slip



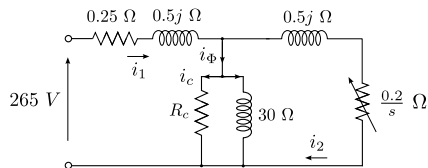
## Exercise 80 - continued

(c-2) Full load current



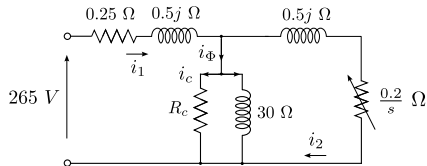
## Exercise 80 - continued

(d) Full load torque



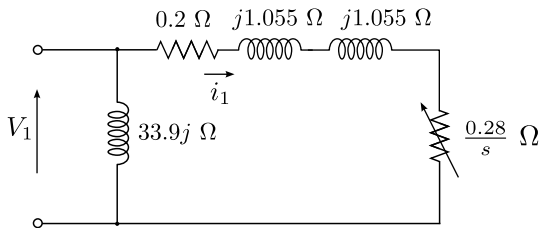
## Exercise 80 - continued

(e) The resistance that must be connected in the rotor so that the maximum torque occurs at start.



## Exercise 81

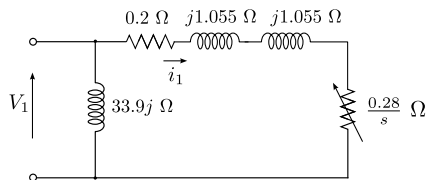
A simplified model of a 3-phase, 150 V per phase, 60 Hz, 6-pole induction motor is shown below.



- Determine the starting torque
- Determine the speed at which the motor drives a 1.8 Nm load.

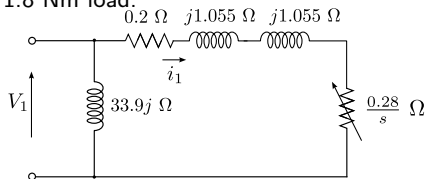
## Exercise 81 - continued

(a) The starting torque



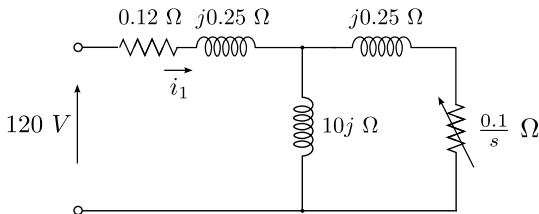
## Exercise 81 - continued

(b) The speed at which the motor drives a  $1.8 \text{ Nm}$  load.



## Exercise 82

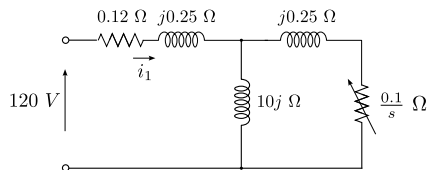
The circuit below represents a single phase of a 3-phase, 60-Hz, 14.5 kW, four-pole induction motor. The rotational loss is 400 W. For 5% slip, determine:



- The rotor speed and current
- The air gap power
- The developed torque

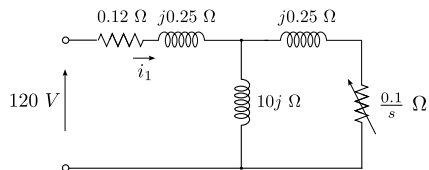
## Exercise 82 - continued

(a) The rotor speed and current



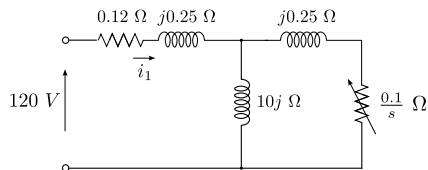
## Exercise 82 - continued

(b) The air gap power



## Exercise 82 - continued

(c) The developed torque



## Quiz

**Q1** - Which of the following does not affect the speed of the induction motor?

- (a) The supply frequency
- (b) The supply voltage
- (c) The rotor resistance
- (d) The magnetizing inductance
- (e) The stator resistance

## Quiz

**Q2** - Which of the following statements is true regarding an induction machine?

- (a) The starting torque is high
- (b) The starting slip is low
- (c) The maximum torque depends on the rotor resistance
- (d) The starting current is higher than the rated current
- (e) All statements are false

## Quiz

**Q3** - A 6 pole 60 Hz induction motor normally runs at which of the following speed?

- (a) 1240 rpm
- (b) 1500 rpm
- (c) 1220 rpm
- (d) 1140 rpm
- (e) 360 rpm

## Quiz

**Q4** - When the supply voltage of an induction motor falls by 3% by which percentage does the torque change?

- (a) 0%
- (b) 3%
- (c) 6%
- (d) 9%
- (e) 12%

## Quiz

**Q5** - In an induction machine, the frequency of the currents flowing in the short-circuited stator winding will be:

- (a) The slip frequency
- (b) The supply frequency
- (c) The frequency of the rotor speed
- (d) Zero
- (e) The synchronous frequency

## Next class...

- Synchronous motors

Additional supporting materials for Lecture 17:

Linear induction motors: <https://goo.gl/ayW1YF>

Induction motors for kids: <https://goo.gl/ME71yS>

Induction motors start torque: <https://youtu.be/JPn50u-N0b0>