

METE 3100U
Actuators and Power Electronics

Lecture 15
Torque Control of DC Motors

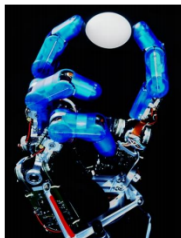
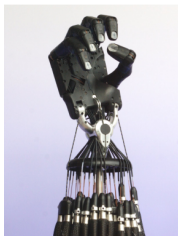
Outline of Lecture 15

By the end of today's lecture, you should be able to

- Implement a closed-loop torque control of a DC motor
- Analyse the influence of motor parameters on the frequency response

Applications

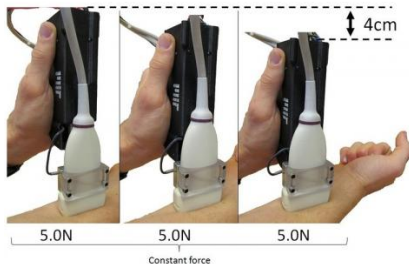
How can the robotic hand actuated by a DC motor be controlled to grasp objects without damaging them?



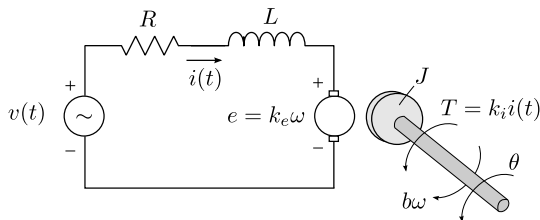
Applications

The quality of ultrasound images depends on the acoustic contact between the probe and the skin.

How can the device be controlled to apply a constant force ?



DC motor model - From lecture 13



$$V(s) = (R + Ls)I(s) + \omega(s)k_m \rightarrow I(s) = \frac{V(s) - \omega(s)k_m}{Ls + R}$$

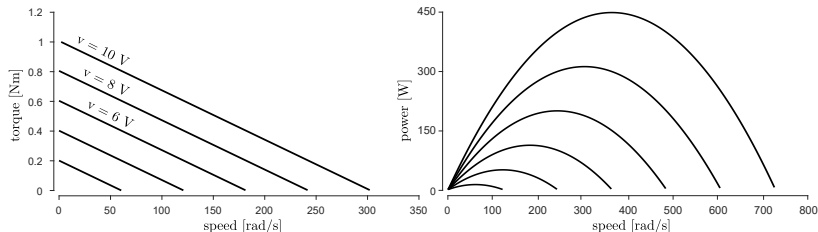
$$T(s) = (Js + b)\omega(s) + T_d(s) \rightarrow \omega(s) = \frac{I(s)k_i - T_d(s)}{Js + b}$$

Under steady-state condition:

$$\omega = \frac{k_i V - RT_d}{k_i k_m + Rb},$$

$$T = \frac{k_i(Vb + k_m T_d)}{k_m k_i + Rb}$$

Speed vs torque characteristics



Suppose steady-state with no load $T_d = 0$ and friction $b \neq 0$:

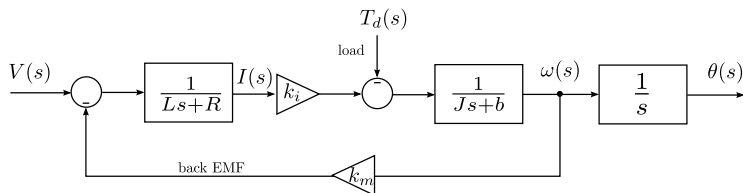
$$\omega < \frac{V}{k_m}, \quad I = \frac{V - \omega k_m}{R}, \quad T = b\omega$$

EMF does not balance applied voltage, and thus $i > 0$

→ Load torque equals frictional torque $T_0 = b\omega$

→ No load current $i = \frac{T_0}{k_i}$

Transfer functions



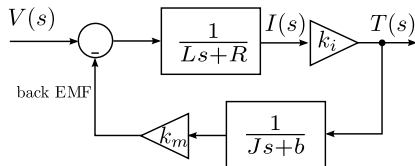
Speed to voltage transfer function for $T_d(s) = 0$

$$S(s) = \frac{\omega(s)}{V(s)} = \frac{k_i}{(Ls + R)(Js + b) + k_i k_m} \quad (1)$$

Position to voltage transfer function for $T_d(s) = 0$

$$P(s) = \frac{\theta(s)}{V(s)} = \frac{k_i}{s[(Ls + R)(Js + b) + k_i k_m]} \quad (2)$$

Transfer functions



Torque to voltage transfer function for $T_d(s) = 0$.

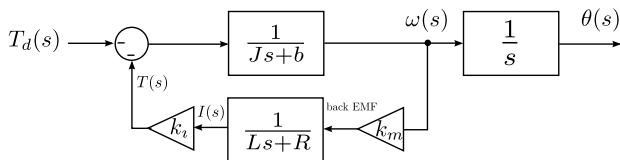
$$M(s) = \frac{T(s)}{V(s)} = \frac{\frac{k_i}{Ls+R}}{1 + \frac{k_m}{Js+b} \frac{k_i}{Ls+R}}$$

$$M(s) = \frac{T(s)}{V(s)} = \frac{k_i(Js + b)}{(Js + b)(Ls + R) + k_i k_m} =$$

The steady-state torque is

$$T = \frac{k_i b}{bR + k_i k_m} V \quad (3)$$

Transfer functions



Load torque $T_d(s)$ to speed transfer function for $V(s) = 0$.

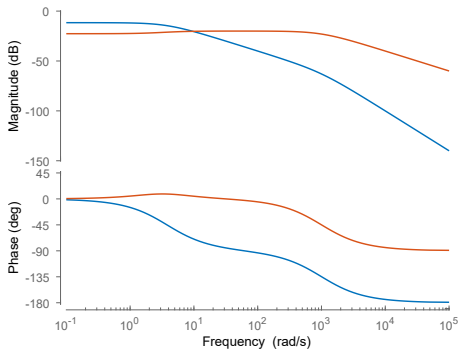
$$L(s) = \frac{\omega(s)}{T_d(s)} = \frac{Ls + R}{(Js + b)(Ls + R) + k_i k_m} = \quad (4)$$

In steady-state:

$$\omega = \frac{R}{bR + k_i k_m} T_d \quad (5)$$

Frequency response

Torque and speed frequency response



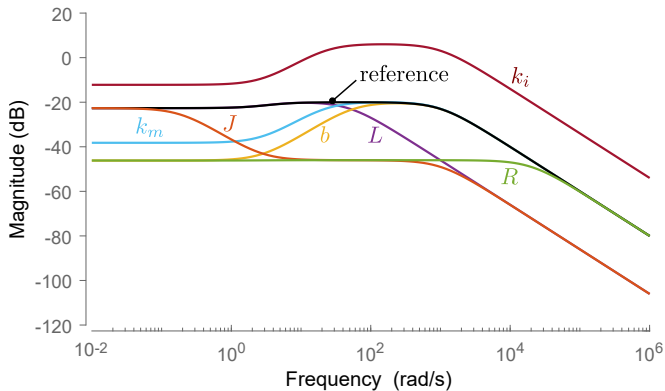
Simulation parameters

k_i	1	Nm/A
k_m	1	V/(rad/s)
R	10	Ω
L	0.01	H
J	0.1	Kg·m ²
b	0.28	Nm/(rad/s)

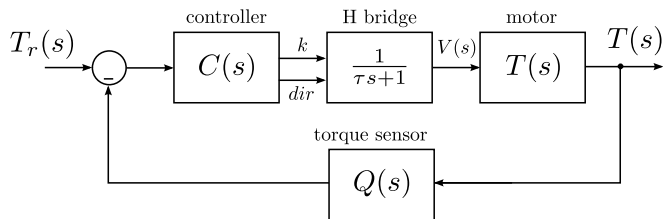
Why is frequency response important?

Frequency response

Torque frequency response as the indicated parameter is increased by a factor of 20 compared to the reference curve.



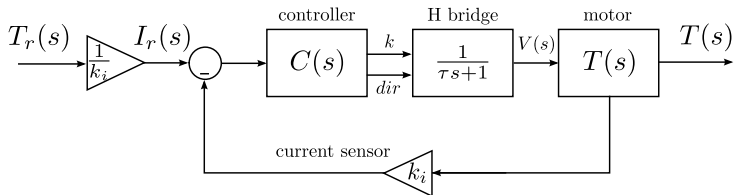
Torque control



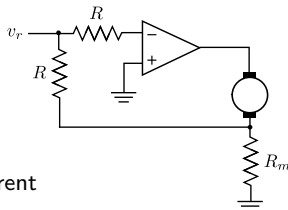
- The H-bridge is modelled as a low pass filter with $\tau \ll \tau_{motor}$
- $Q(s)$ is the **torque** sensor transfer function
- $C(s)$ is the controller. For a PID:

$$C(s) = k_d + k_d s + \frac{k_i}{s} \quad (6)$$

Torque control

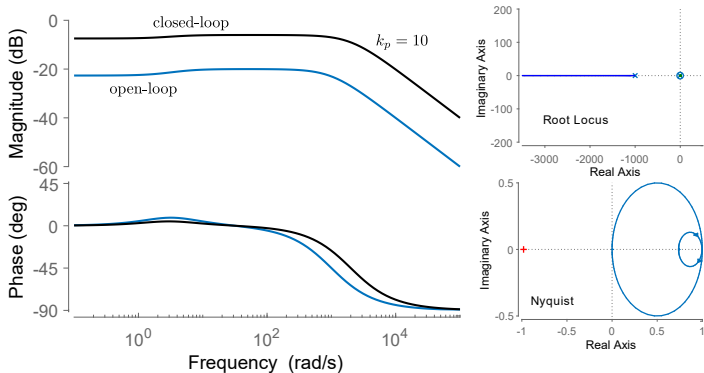


- Current control vs torque control
- The desired current is $i = T_r(s)/k_i$
- A shunt resistance is used to measure the current
- The motor torque is $T(s) =$



Torque control

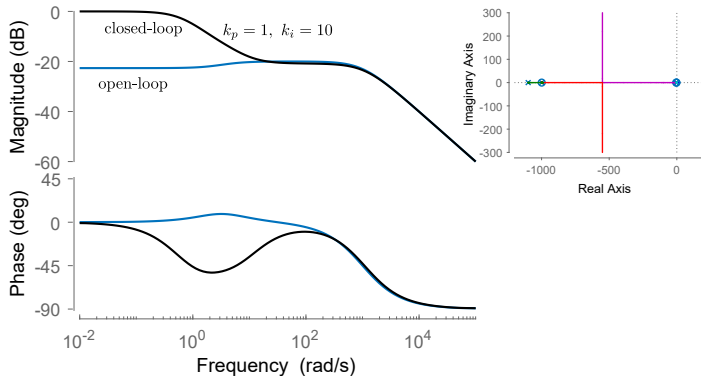
Proportional controller: $k_p \neq 0$, $k_i = k_d = 0$.



- The closed-loop system is stable
- There is no overshoot. What limits k_p ?

Torque control

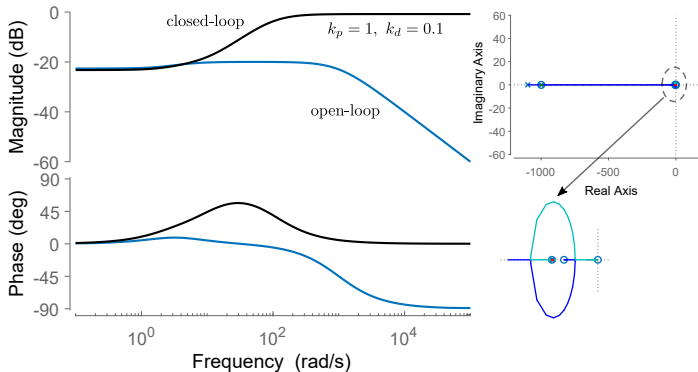
Proportional-integral controller: $k_p = 1$, $k_d = 0$, $k_i = 10$.



- Integral gain leads to overshoot
- The system is always stable

Speed integral

Proportional-derivative controller: $k_p = 1$, $k_i = 0$, $k_d = 0.1$.



→ Very high control gain lead to overshoot

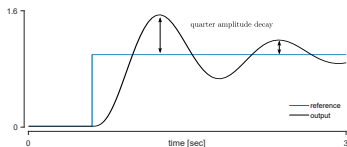
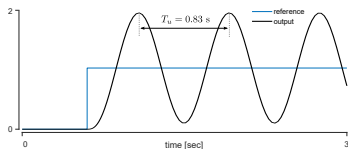
→ The system is allays stable for $k_d > 0$

Manual PID tuning

Step 1: Find the critical proportional gain k_p before instability.

→ Set $k_i = k_d = 0$.

→ Slowly increase k_p to the edge of stability



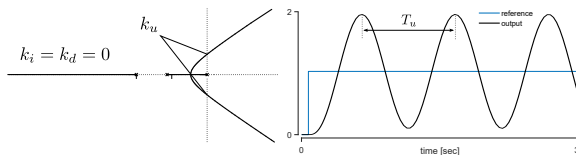
Step 2: Reduce k_p to achieve a step response with approximately a quarter amplitude decay.

→ I.e.: the overshoot drops to 25% of the initial value after one period.

→ As an initial approximation set $k_p = 0.5k_u$.

Step 3: Manually tune k_i and k_p ...

Ziegler-Nichols PID tuning - Method 1

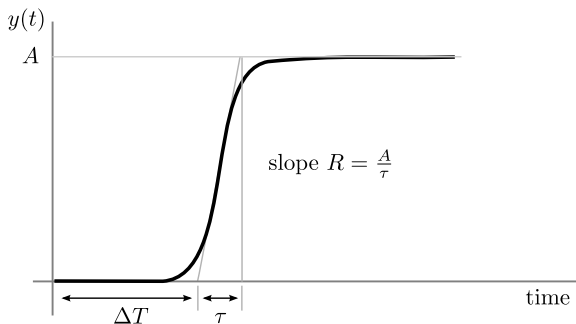


Controller type	k_p	k_i	k_d
Proportional $C(s) = k_p$	$0.5k_u$	0	0
Proportional-integral $C(s) = k_p + k_i s^{-1}$	$0.45k_u$	$\frac{0.54k_u}{T_u}$	0
PID $C(s) = k_p + k_i s^{-1} + k_d s$	$0.6k_u$	$\frac{1.2k_u}{T_u}$	$\frac{0.6k_u T_u}{8}$

Ziegler-Nichols PID tuning - Method 2

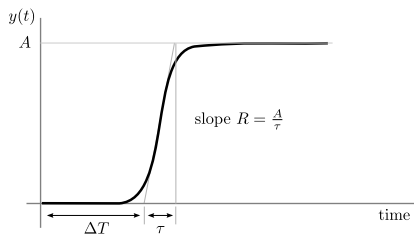
Many systems can be approximated by the step response of

$$\frac{Y(s)}{U(s)} = \frac{Ae^{-\Delta t s}}{\tau s + 1} \quad (7)$$



This is a first order system with a time delay of Δt sec.

Ziegler-Nichols PID tuning - Method 2



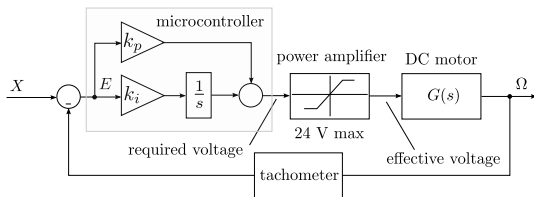
Controller type	k_p	k_i	k_d
Proportional $C(s) = k_p$	$\frac{1}{R\Delta T}$	0	0
Proportional-integral $C(s) = k_p + k_i s^{-1}$	$\frac{0.9}{R\Delta T}$	$\frac{0.27}{R\Delta T^2}$	0
PID $C(s) = k_p + k_i s^{-1} + k_d s$	$\frac{1.2}{R\Delta T}$	$\frac{0.6}{R\Delta T^2}$	$\frac{0.6}{R}$

Integrator anti-windup

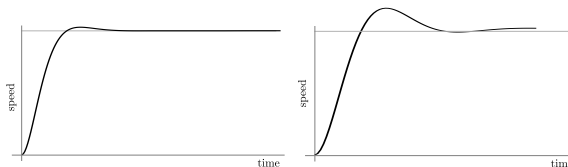
The output of the actuator can saturate.

→ The maximum torque, current, voltage, etc, is reached

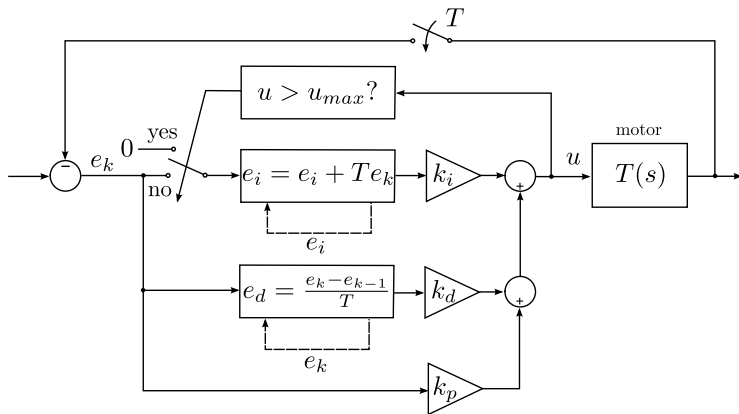
→ The output voltage of a motor speed controller is limited



If the controller is implemented digitally, if $|u| \geq u_{max}$, set $k_i = 0$



Anti-windup implementation



Exercise 70

A Maxom RE-40 DC motor has the characteristics shown in the table.

Torque constant	k_i	32	mNm/A
Speed constant	k_m	32	mV/(rad/s)
Resistance	R	0.3	Ω
Inductance	L	0.082	H
Rotor inertia	J	1.42×10^{-5}	kg·m ²
Friction constant	b	1×10^{-5}	Nm/(rad/s)

Determine:

- The stall torque.
- The developed torque and speed when the input power is 100 W and the load torque is 0.1 Nm.

Exercise 70 - continued

$$\omega = \frac{k_i V - R T_d}{k_i k_m + R b},$$

$$T = \frac{k_i (V b + k_m T_d)}{k_m k_i + R b}$$

(a) The stall torque

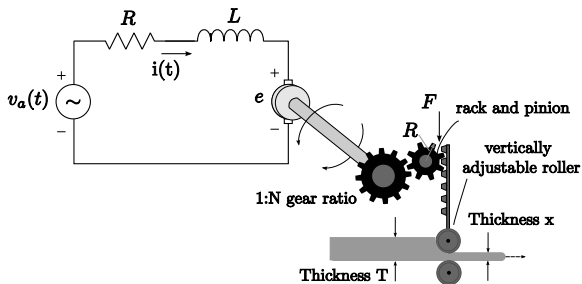
Exercise 70 - continued

$$\omega = \frac{k_i V - R T_d}{k_i k_m + R b}, \quad T = \frac{k_i (V b + k_m T_d)}{k_m k_i + R b}$$

(b) The torque and speed when the input power is 100 W and $T_d = 0.1$ Nm.

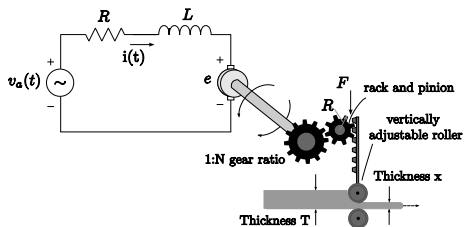
Exercise 71

In the milling rolling mill the motion of the adjustable roller has a damping coefficient b , and the force exerted the rolled material on the roller is proportional to the material's change in thickness: $F_c = c(T - x)$. Suppose that the DC motor as a torque constant k_i and back emf constant k_m . The rack/pinion has a radius R .

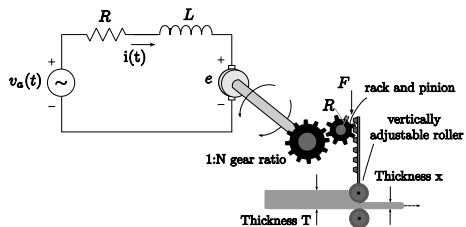


Find the relation between the applied voltage $V_a(s)$ and the material thickness $X(s)$.

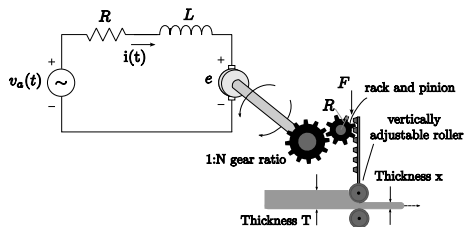
Exercise 71 - continued



Exercise 71 - continued



Exercise 71 - continued



Exercise 72

A DC motor has the characteristics shown in the table below. The motor torque is to be controlled using a closed-loop proportional controller such that there is no overshoot.

Torque constant	k_i	1	Nm/A
Speed constant	k_m	1	V/(rad/s)
Resistance	R	0.28	Ω
Inductance	L	0.01	H
Rotor inertia	J	0.1	kg·m ²
Friction constant	b	0.28	Nm/(rad/s)

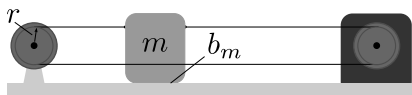
Determine the control gains that meet the above requirement.

Exercise 72 - continued

Exercise 72 - continued

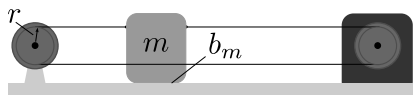
Exercise 73

The DC motor of Exercise 70, is used to control the position of the 0.25 kg mass as shown. The mass is connected to the motor using a pulley-belt mechanism. Each pulley have a radius of 250 mm and the coefficient of viscous friction $b_m = 1 \text{ N}/(\text{m}/\text{s})$.



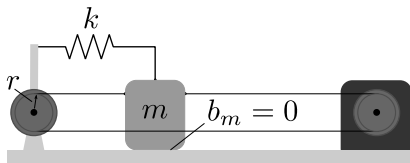
Determine the required electrical power to move the mass at a constant speed of 500 mm/s.

Exercise 73 - continued



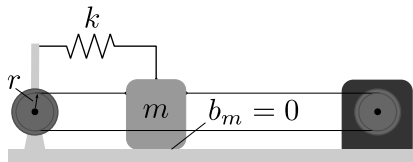
Exercise 74

The DC motor of Exercise 70, is used to control the position of the 0.25 kg mass that is connected to a spring having stiffness $k = 10 \text{ N/m}$. The mass is connected to the motor using a pulley-belt mechanism. Each pulley has a radius of 250 mm.



Determine the required motor input voltage to displace the mass by 150 mm.

Exercise 74 - continued



Next class...

- Synchronous motors

Additional supporting materials for Lecture 15:

DC motor Matlab model: <https://bit.ly/2Szf1Mv>

DC motor control in Arduino: <https://bit.ly/2Dgh3Yq>

DC motor with H-bridge: <https://bit.ly/2RMkoDJ>