METE 3100U
Actuators and Power Electronics

Lecture 14

Speed and Position Control of DC Motors

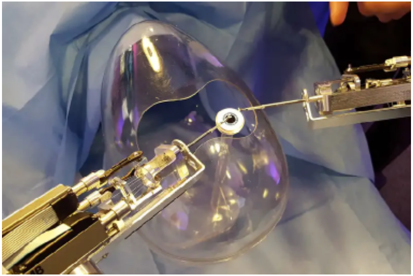
Outline of Lecture 14

By the end of today's lecture, you should be able to

- Implement a closed-loop speed control of a DC motor
- Implement a closed-loop position control of a DC motor
- Analyse the influence of motor parameters on the frequency response

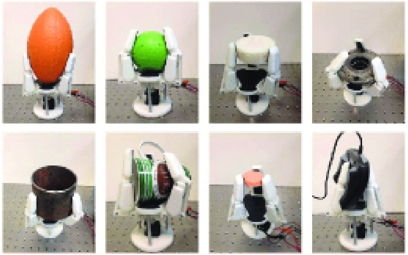
Applications

Robot surgeon can slice eyes finely enough to remove cataracts. How are the motors controlled to achieve micrometer precision?

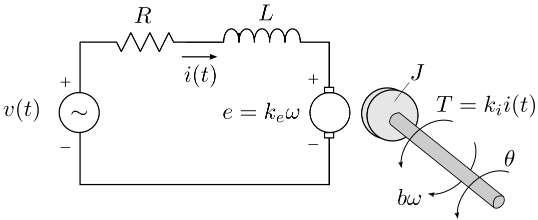


Applications

How can the robotic hand actuated by a DC motor be controlled to grasp objects without damaging them?



DC motor model - From lecture 13



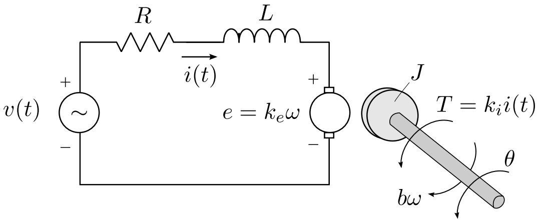
$$V(s) = (R + Ls)I(s) + \omega(s)k_m \rightarrow I(s) = \frac{V(s) - \omega(s)k_m}{Ls + R}$$

$$T(s) = (Js + b)\omega(s) + T_d(s) \rightarrow \omega(s) = \frac{I(s)k_i - T_d(s)}{Js + b}$$

Under steady-state condition:

$$I = \frac{V - \omega k_m}{R} \qquad \omega = \frac{I \times k_i - T_d(s)}{b}$$

DC motor model - From lecture 13



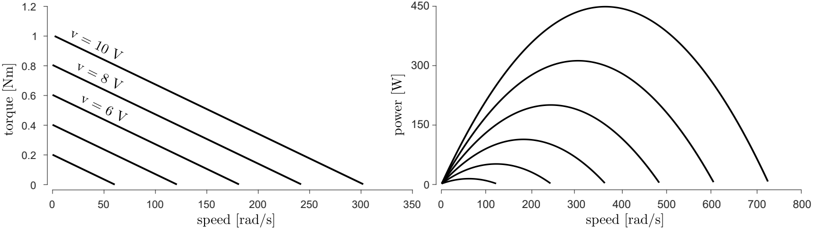
$$I = \frac{V - \omega k_m}{R} \qquad \omega = \frac{I \times k_i - T_d(s)}{b}$$

If the motor is driven at a constant voltage $V(t) = V$, the steady state speed and torque satisfy

$$\omega = \frac{k_i V - RT_d}{k_i k_m + Rb}, \qquad T = \frac{k_i(Vb + k_m T_d)}{k_m k_i + Rb}$$

Speed *vs* torque characteristics

$$\omega = \frac{k_i V - R T_d}{k_i k_m + Rb}, \qquad T = \frac{k_i(Vb + k_m T_d)}{k_m k_i + Rb}$$

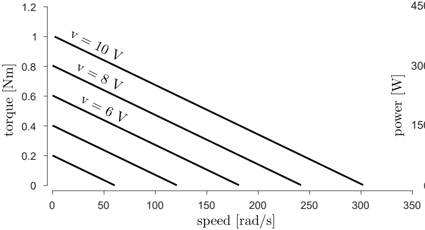


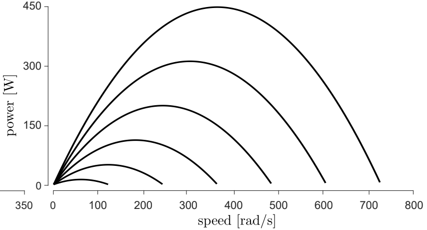
Suppose steady-state with no load $T_d = 0$ and no friction $b = 0$:

$$\omega = \frac{V}{k_m}, \qquad T =$$

EMF balances applied voltage, and thus $i =$

Speed *vs* torque characteristics





Suppose steady-state with a load $T_d \neq 0$ and friction $b \neq 0$:

$$\omega < \frac{V}{k_m}, \qquad I = \frac{V - \omega k_m}{R}, \qquad\qquad T = ?$$

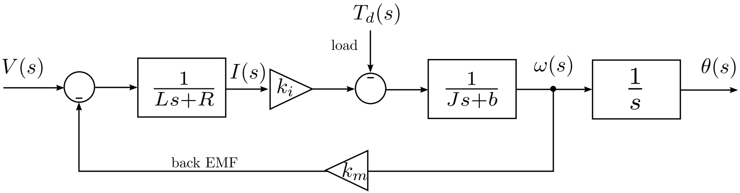
EMF does not balance applied voltage, and thus $i > 0$

→ Speed and torque depend on load and friction

→ Friction is always present

→ Load torque is typically varying and unknown

Transfer functions



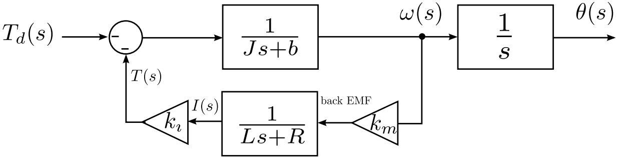
Speed to voltage transfer function for $T_d(s) = 0$

$$S(s) = \frac{\omega(s)}{V(s)} = \frac{k_i}{(Ls+R)(Js+b)+k_i k_m} \tag{1}$$

Position to voltage transfer function for $T_d(s) = 0$

$$P(s) = \frac{\theta(s)}{V(s)} = \frac{k_i}{s[(Ls+R)(Js+b)+k_i k_m]} \tag{2}$$

Transfer functions



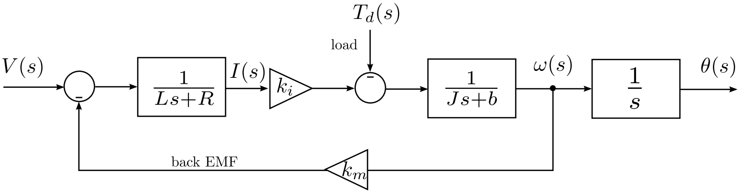
Load torque $T_d(s)$ to speed transfer function for $V(s) = 0$.

$$L(s) = \frac{\omega(s)}{T_d(s)} = \frac{Ls + R}{(Js + b)(Ls + R) + k_i k_m} \quad (3)$$

In steady-state:

$$\omega = \frac{R}{bR + k_i k_m} T_d \quad (4)$$

Superposition



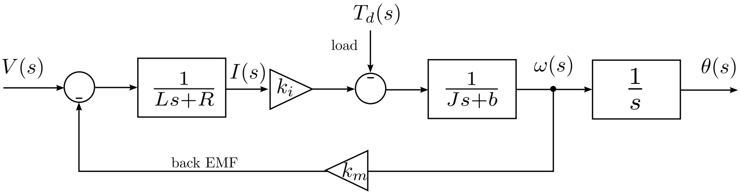
The effects of the load and voltage can be added to find the system function

$$S(s) = \frac{\omega(s)}{V(s)} = \frac{k_i}{(Ls+R)(Js+b)+k_ik_m} \tag{5}$$

$$L(s) = \frac{\omega(s)}{T_d(s)} = \frac{Ls+R}{(Js+b)(Ls+R)+k_ik_m} \tag{6}$$

$$\omega(s) = \frac{k_i}{(Ls+R)(Js+b)+k_ik_m}V(s) - \frac{Ls+R}{(Js+b)(Ls+R)+k_ik_m}T_d(s) \tag{7}$$

Superposition



The effects of the load and voltage can be added to find the system function

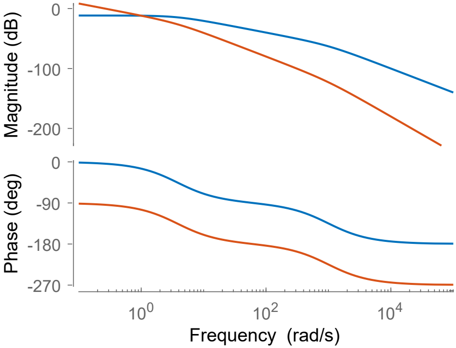
$$P(s) = \frac{\theta(s)}{V(s)} = \frac{k_i}{s[(Ls+R)(Js+b)+k_ik_m]} \quad (8)$$

$$N(s) = \frac{\theta(s)}{T_d(s)} = \frac{Ls+R}{s[(Js+b)(Ls+R)+k_ik_m]} \quad (9)$$

$$\theta(s) = \frac{k_i}{s[(Ls+R)(Js+b)+k_ik_m]}V(s) - \frac{Ls+R}{s[(Js+b)(Ls+R)+k_ik_m]}T_d(s) \quad (10)$$

Frequency response

Position and speed frequency response



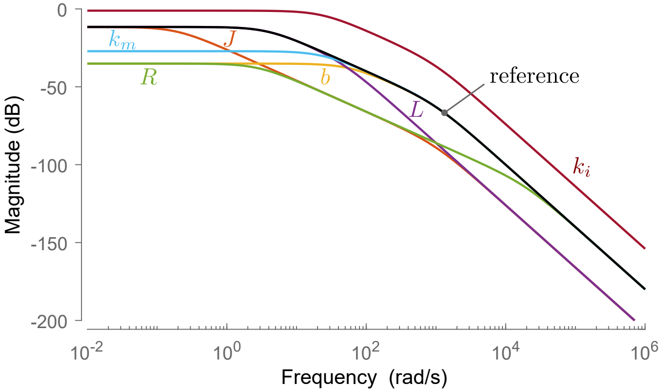
Simulation parameters

k_i	1	Nm/A
k_m	1	V/(rad/s)
R	10	Ω
L	0.01	H
J	0.1	kg·m^2
b	0.28	Nm/(rad/s)

Why is frequency response important?

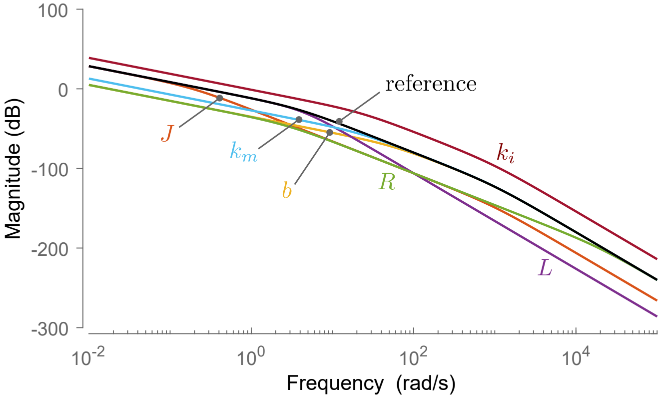
Frequency response

Speed frequency response as the indicated parameter is increased by a factor of 20 compared to the reference curve.

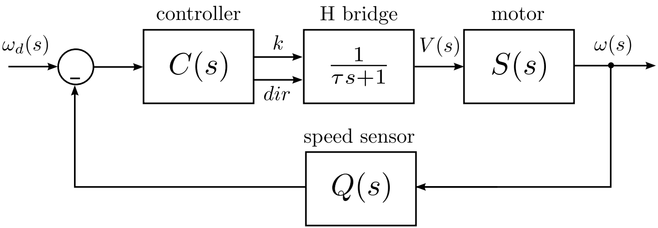


Frequency response

Position frequency response as the indicated parameter is increased by a factor of 20 compared to the reference curve.



Speed control



→ The H-bridge is modelled as a low pas filter with $\tau << \tau_{motor}$

→ $Q(s)$ is the sensor transfer function

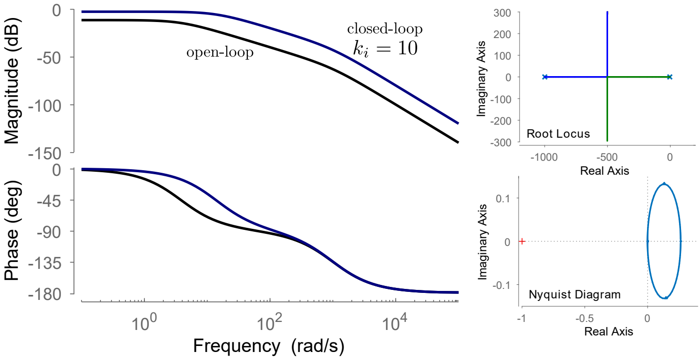
→ $C(s)$ is the controller. For a PID:

$$C(s) = k_d + k_d s + \frac{k_i}{s} \tag{11}$$

→ How does the controller affect the frequency response?

Speed control

Proportional controller: $k_p \neq 0,\ k_i = k_d = 0.$

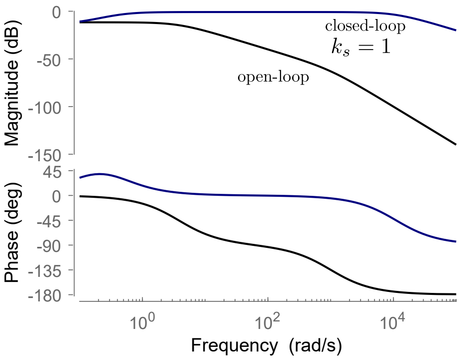


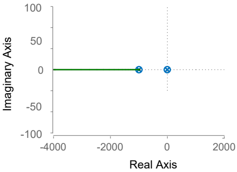
→ Overshoot is present for high control gains

→ The system is stable $\forall\ k_p > 0$

Speed control

Proportional-derivative controller: $k_p \neq 0,\ k_d \neq 0,\ k_i = 0.$



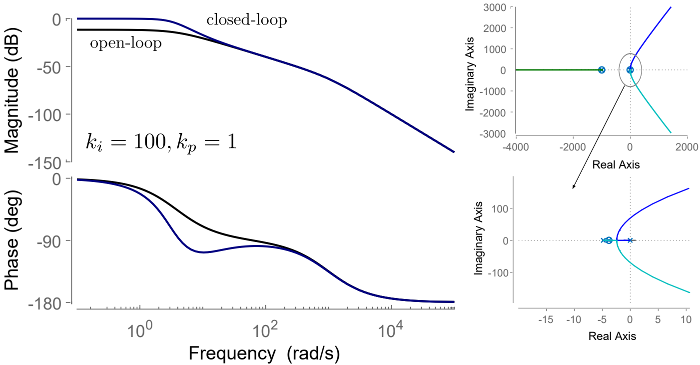


→ Derivative gain eliminates overshoot

→ The system is always stable

Speed control

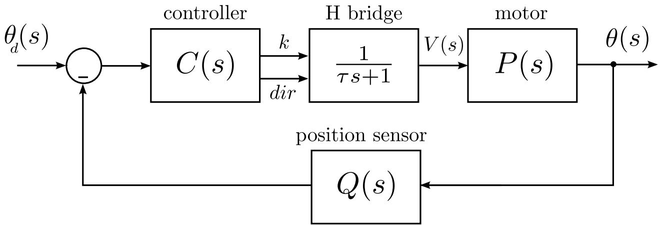
Proportional-integral controller: $k_p \neq 0,\ k_i \neq 0,\ k_d = 0.$



→ Overshoot is present for high control gains

→ The system may become unstable for high k_i

Position control



→ The H-bridge is modelled as a low pas filter with $\tau << \tau_{motor}$

→ $Q(s)$ is the sensor transfer function

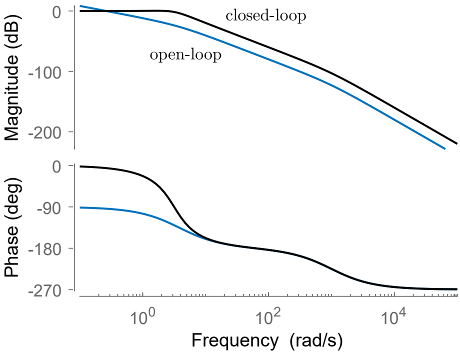
→ $C(s)$ is the controller. For a PID:

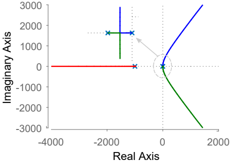
$$C(s) = k_d + k_d s + \frac{k_i}{s} \tag{12}$$

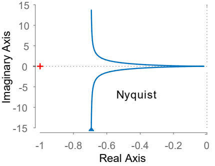
→ Recall that $P(s) = \frac{S(s)}{s}$

Position control

Proportional controller: $k_p \neq 0,\ k_i = k_d = 0.$





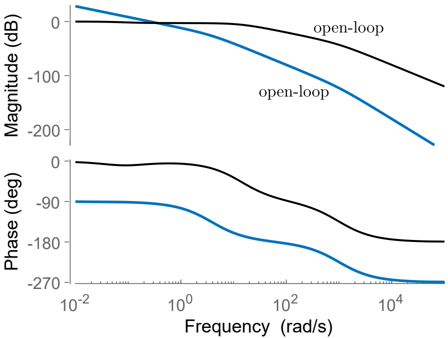


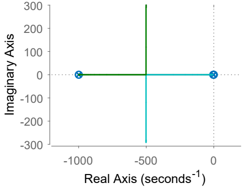
$\rightarrow$ Overshoot is present for low control gains

$\rightarrow$ The system is unstable for $k_p >> 0$

Position control

Proportional-derivative controller: $k_p = 1,\ k_d = 10,\ k_i = 0.$



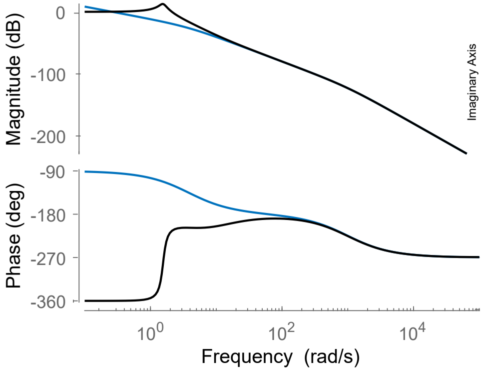


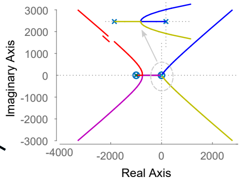
→ Overshoot is present for low control gains

→ The system is stable for $k_p = 1,\ k_d > 0$

Position control

Proportional-integral controller: $k_p = 1$, $k_d = 0$, $k_i = 1$.

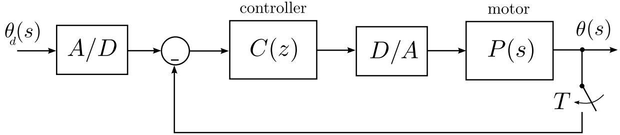




→ Overshoot is present for low control gains

→ The system is unstable for high values of k_i

PID controller implementation

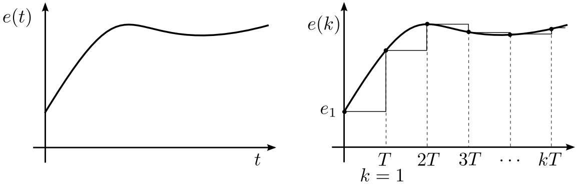


Digital PID control algorithm

⇒ Error is sampled every T seconds

⇒ Output is updated every T seconds

⇒ Analog PID discretized into the differential equation



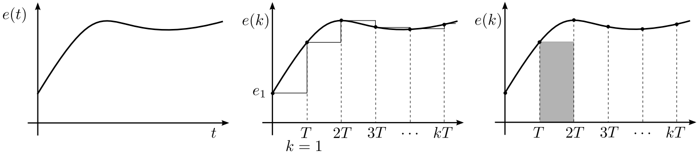
PID controller implementation

Analogue (continous) PID controller

$$C(s) = k_p + \frac{k_i}{s} + k_d s$$

The control output is

$$Z(s) = E(s)C(s) \tag{13}$$



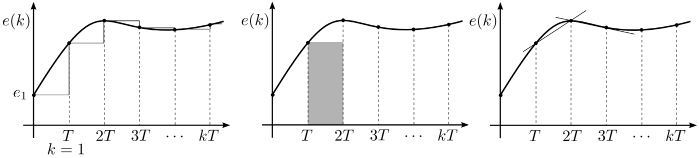
The integral term is

$$\int_0^t e(t)dt \approx T\sum_{i=0}^{k} e(i) \tag{14}$$

PID controller implementation

The derivative term is

$$\frac{de(t)}{dt} \approx \frac{e(i) - e(i-1)}{T} \tag{15}$$

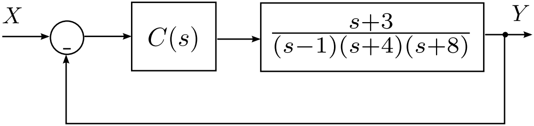


In discrete form the PID is

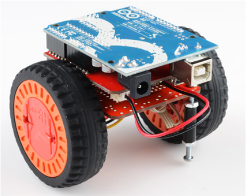
$$Z(i) = k_i \left(T \sum_{i=0}^{k} e(i) \right) + k_d \left(\frac{e(i) - e(i-1)}{T} \right) + k_d e(i) \tag{16}$$

Exercise 66

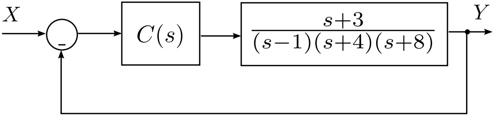
The 2 wheel robot is actuated by a single DC motor that is used to balance the robot using a *PI* controller $G(s)$ have the same k_i and k_p gains.



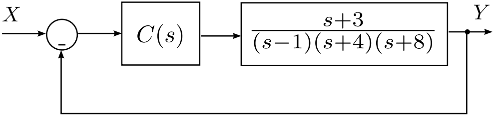
Select the controller gain $k_p = k_i$ so that all of the complex poles have a damping ratio higher than 0.6.



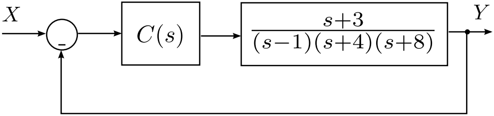
Exercise 66 - continued



Exercise 66 - continued

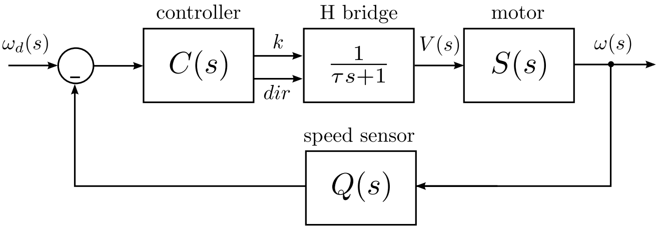


Exercise 66 - continued



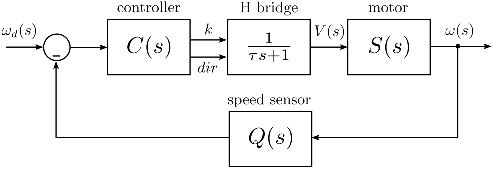
Exercise 67

Write an Arduino code of a PID controller that can be used to regulate the speed of a DC motor following the arrangement shown.



The PID Library is available in the Arduino IDE Library Manager.

Exercise 67 - continued



Exercise 68

A 24V, Maxom RE-40 DC motor has the characteristics shown in the table.

Torque constant	k_i	32	mNm/A
Speed constant	k_m	32	mV/(rad/s)
Resistance	R	0.3	Ω
Inductance	L	0.082	H
Rotor inertia	J	1.42×10^{-5}	kg·m^2
Friction constant	b	1×10^{-5}	Nm/(rad/s)

Determine:

(a) The no-load speed

(b) The no-load torque and current

(c) The voltage and current required to drive a 0.1 Nm load at 5 rad/s.

Exercise 68 - continued

$$\omega = \frac{k_i V - RT_d}{k_i k_m + Rb}, \qquad T = \frac{k_i(Vb + k_m T_d)}{k_m k_i + Rb}$$

(a) The no-load speed

Exercise 68 - continued

$$\omega = \frac{k_i V - RT_d}{k_i k_m + Rb}, \qquad T = \frac{k_i(Vb + k_m T_d)}{k_m k_i + Rb}$$

(b) The no-load torque and current

Exercise 68 - continued

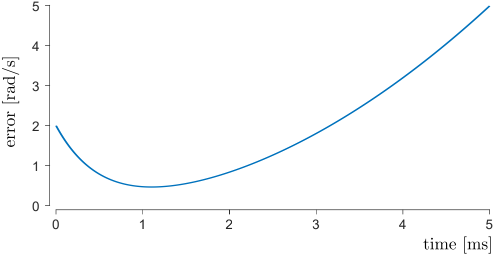
$$\omega = \frac{k_i V - RT_d}{k_i k_m + Rb}, \qquad T = \frac{k_i(Vb + k_m T_d)}{k_m k_i + Rb}$$

(c) The voltage and current required to drive a 0.1 Nm load at 5 rad/s.

Exercise 69

The measured speed tracking error of a PID-controlled DC motor during the interval shown is described by

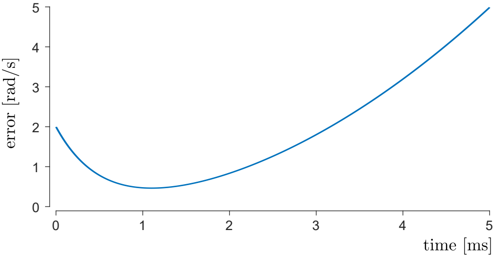
$$e(t) = \frac{t^2}{5} + 2e^{-2t}$$



where t is in ms. If the PID is implemented in a microcontroller running with a sampling frequency of 1kHz, calculate the controller output when $t = 5$ ms.

Exercise 69 - continued

$$e(t) = \frac{t^2}{5} + 2e^{-2t}$$



Exercise 69 - continued

Next class...

- Torque control of DC motors

Additional supporting materials for Lecture 14:

Simulink - DC motor position control: `https://goo.gl/WKAaW1`

Simulink - DC motor seed control: `https://goo.gl/4yCZ2B`

Speed Control using an H-Bridge: `https://goo.gl/gV8T5x`