METE 3100U
Actuators and Power Electronics

Lecture 13
DC Machines

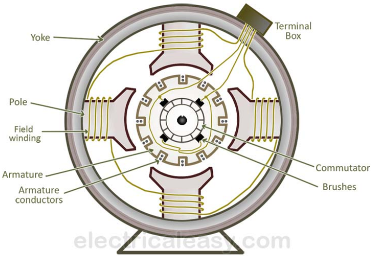
Outline of Lecture 13

By the end of today's lecture, you should be able to

- Understand the working principle of DC machines
- Analyse and understand different architectures
- Model a DC machine

Applications

A DC generator converts mechanical energy into direct current electricity based on the principle of production of dynamically induced emf.



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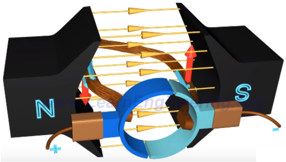
Applications

A brushed DC motor is an internally commutated electric motor designed to be run from a direct current power source.

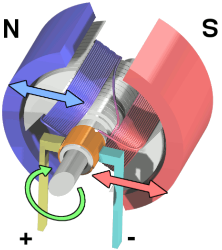
How is the current reversed to create a continuous rotation?



Applications

In stepper motors the orientation of the magnetic field is controlled digitally.

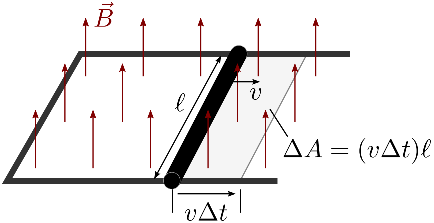
DC motors have mechanical commutators.



Electromotive force in a moving wire

The emf induced in a conductor moving in a magnetic field is

$$\epsilon = -N\frac{d\Phi}{dt} = \frac{\Delta\Phi}{\Delta t} \tag{1}$$



$\Delta\Phi$ as a function of the area is

$$\Delta\Phi = B(\Delta A) \tag{2}$$

Thus, the emf is:

$$\epsilon = -N\frac{d\Phi}{dt} = \frac{B(\Delta A)}{\Delta t} = \frac{B(v\Delta t\ell)}{\Delta t} = \ell vB \tag{3}$$

Emf will induce a current if the loop is closed.

Lorentz force

A particle of charge q moving with a velocity $\vec{v}$ in a electric field $\vec{E}$ and a magnetic field $\vec{B}$ experiences a force

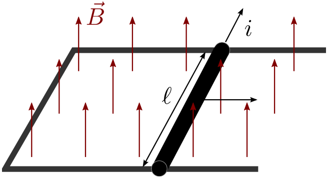
$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{4}$$

Force on a current (i) carrying wire

$$\vec{F} = i \int \vec{d\ell} \times \vec{B} \tag{5}$$

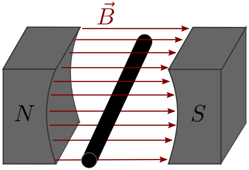
In the case of a straight wire

$$\vec{F} = i\vec{\ell} \times \vec{B} \tag{6}$$

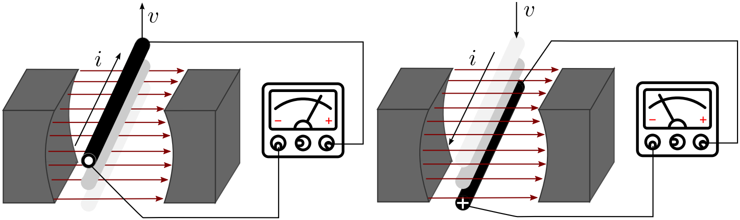


DC generator

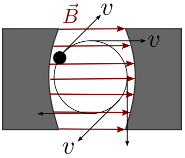
Consider a wire moving in a constant magnetic field $\vec{B}$

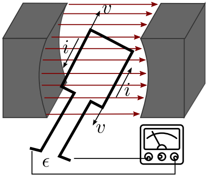


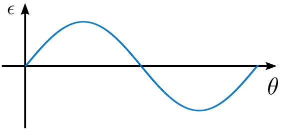
The induced emf changes with the direction of the velocity



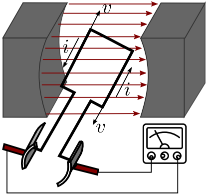
Commutator - voltage rectification

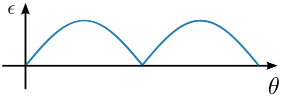






The angle between $\vec{v}$ and $\vec{B}$ is not constant

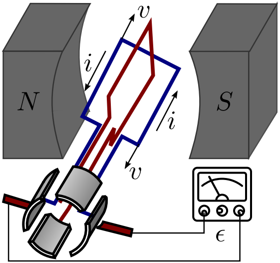


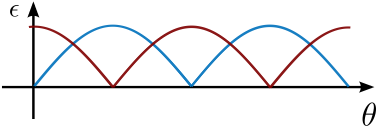


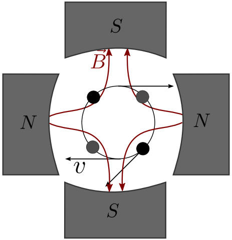
$$\epsilon = \frac{d\Phi}{dt} = \frac{-dBA\cos\theta}{dt}$$

$$\epsilon = \ell v B \sin(\theta)$$

Commutator - voltage rectification







The induced voltage depends on:

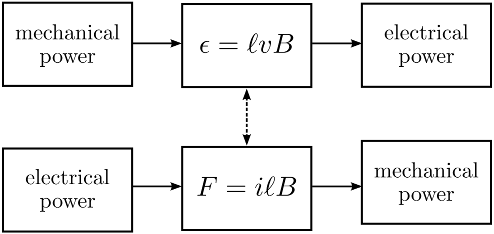
→ The intensity of the magnetic field B

→ The length of the conductor ℓ

→ The angular speed of the conductor v

DC generator *vs* DC motor

DC motors and generators operate based on the same principle



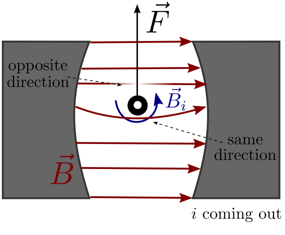
$\Rightarrow$ Motors have a DC current as an input and output mechanical power

$\Rightarrow$ Generators has a mechanical energy as an input and output DC current

DC motor

A conductor carrying a current i experiences a force

$$\vec{F} = i\vec{\ell} \times \vec{B}$$

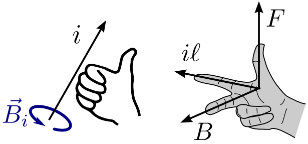


The force depends on

→ The intensity and direction of the magnetic field

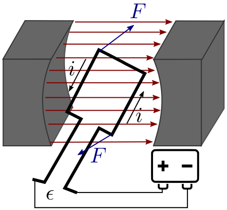
→ The length of the wire

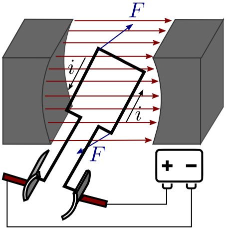
→ The intensity and direction of the current



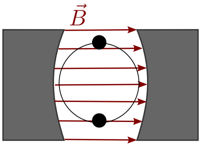
DC motor

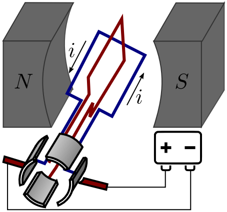
The force on each segment has opposite directions





Mutiple conductors are typically use to smoothen the force



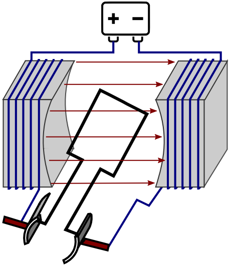


Series *vs* shunt windings

Electromagnets may be used instead of permanent magnets.

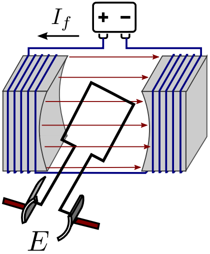
⇒ Series wound: The electromagnet and rotor coils are connected in series.

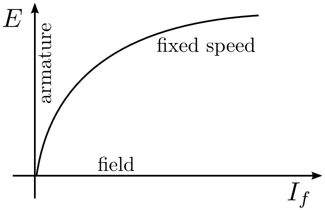
⇒ Shunt wound: The electromagnet and robot coils are connected in parallel.



Magnetisation curve

The magnetic circuit is prone to saturation of the core





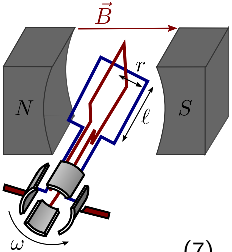
Experimentally:

⇒ Rotate the machine at a constant speed

⇒ Change the current in the winding circuit

⇒ Measure E at the open circuit armature terminal

Armature voltage



The voltage induced in the winding is

$$e_t = 2\ell B(\theta)\omega r$$

For an armature with p poles, the area of each pole is

$$A = \frac{2\pi r\ell}{p} \tag{7}$$

and since $B = \Phi/A = \Phi p/(2\pi r\ell)$, the emf is

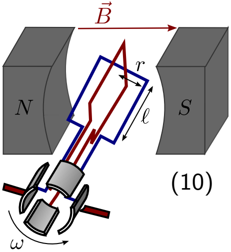
$$e_t = \frac{\Phi p}{\pi}\omega \tag{8}$$

If the armature winding has N tuns

$$E = \frac{N}{p}e_t = \frac{N}{p}\frac{\Phi p}{\pi}\omega = \underbrace{\frac{N}{\pi}}_{k_a}\Phi\omega \tag{9}$$

$k_a = N/\pi$ is the armature constant.

Developed torque



The force developed in on the conductor is

$$f = B(\theta)\ell i_c = B(\theta)\ell \frac{I_a}{p} \tag{10}$$

where I_a is the terminal current. The torque is

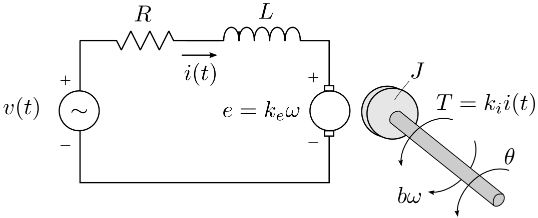
$$T_c = f_c r = \frac{\Phi p I_a}{2\pi p} = \frac{\Phi I_a}{2\pi} \tag{11}$$

For N conductors:

$$T = 2NT_c = \frac{N\Phi}{\pi} = K_a \Phi I_a \tag{12}$$

Conversation of energy $P_e = P_m$:

Electromechanical modelling

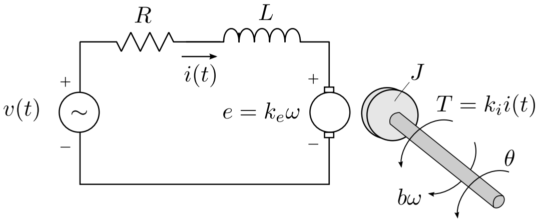


Electric circuit:

$$v(t) = i(t)R + L\frac{d}{dt}i(t) + e$$
$$e(t) = K_m\omega(t)$$

Transfer function

Electromechanical modelling



Mechanical characteristics

$$\sum T = J\dot{\omega}(t)$$

$$T(t) - b\omega - T_d(t) = J\dot{\omega}(t)$$

$$T(t) = K_i i(t)$$

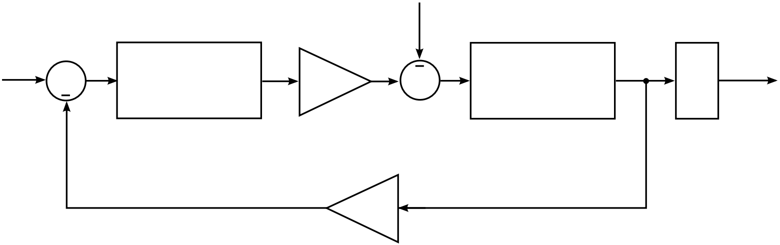
Transfer function

Electromechanical modelling

$$V(s) = (R + Ls)I(s) + \omega(s)k_m \rightarrow I(s) = \frac{V(s) - \omega(s)k_m}{Ls + R}$$

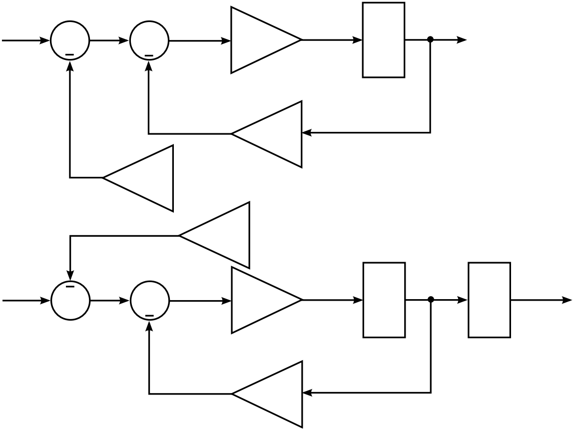
$$T(s) = (Js + b)\omega(s) + T_d(s) \rightarrow \omega(s) = \frac{I(s)k_i - T_d(s)}{Js + b}$$

Block diagram



Electromechanical modelling

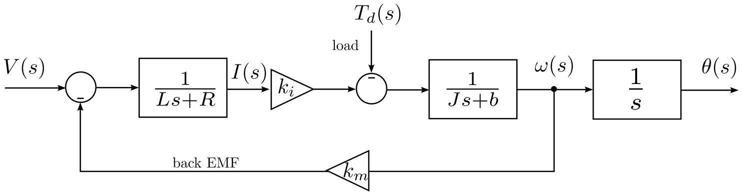
$$V(s) - I(s)R - \omega(s)k_m = LI(s)s$$
$$T(s) - T_d(s) - \omega(s)b = J\omega(s)s$$



Electromechanical modelling

Stall current: The maximum current drawn (maximum torque).

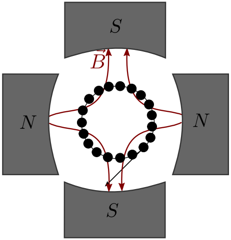
No-load current: The current drawn when operating with no load



$$G(s) \approx \frac{\theta(s)}{V(s)} = \frac{k_i}{s[R(Js+b)+k_i k_m]} \tag{13}$$

Exercise 59

A 4-pole DC machine has an armature radius of 125 mm, and a length of 250 mm. The poles cover 75% of the armature periphery. The windings consist of 33 coils accommodated in 33 slots, each having 7 turns. The average flux density under each pole is 0.75 T.



Calculate:

(a) The armature constant K_a.

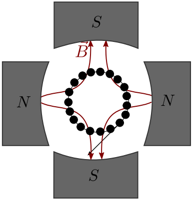
(b) The induced emf if $\omega = 1000$ rpm.

(c) The torque when the armature current is 400 A.

(d) The power developed by the armature.

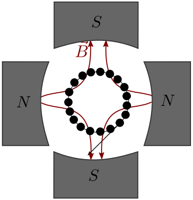
Exercise 59 - continued

(a) The armature constant K_a.



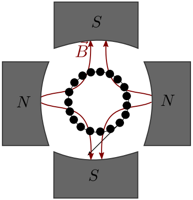
Exercise 59 - continued

(b) The induced emf if $\omega = 1000$ rpm.



Exercise 59 - continued

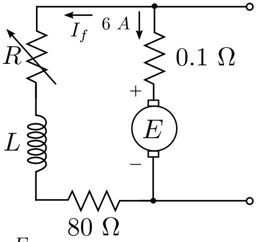
(c) The torque when the armature current is 400 A.



(d) The power developed by the armature.

Exercise 60

The DC machine shown is connected to a 100 V DC supply and is operated as a DC shunt motor. At no-load, the motor runs at 1000 rpm and the armature takes 6 Amp. A potentiometer R is used to regulate the speed.



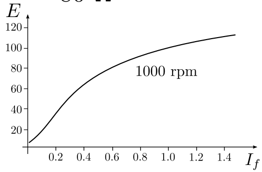
Calculate:

(a) The value of the variable resistor R.

(b) The mechanical power at 1000 rpm.

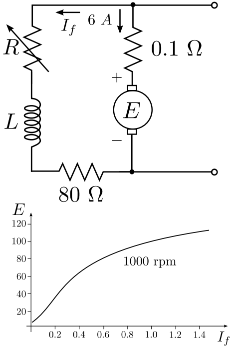
(c) The speed, torque, and efficiency when $I_a = 120$ V

(d) The stall torque.



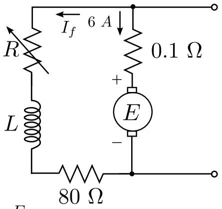
Exercise 60 - continued

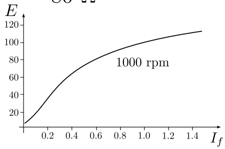
(a) The value of the variable resistor R.



Exercise 60 - continued

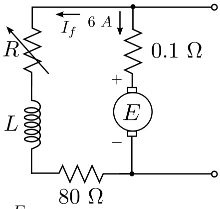
(b) The mechanical power at 1000 rpm.

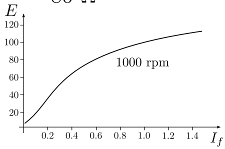




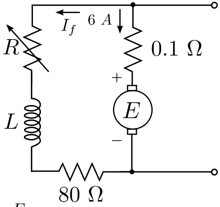
Exercise 60 - continued

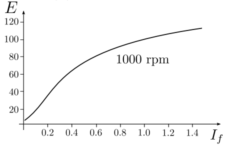
(c) The speed and torque when $I_a = 120$ A





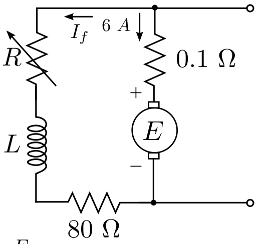
Exercise 60 - continued

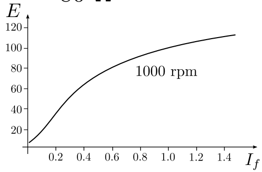




Exercise 61

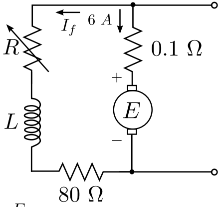
The DC 12kW, 100 V, 1000 rpm shunt generator shown has 1200 turns per pole and is operated as a separately exited DC generator at 1000 rpm. The rated field current is 1 Amp.

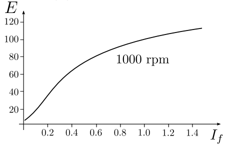




Determine the terminal voltage at full load.

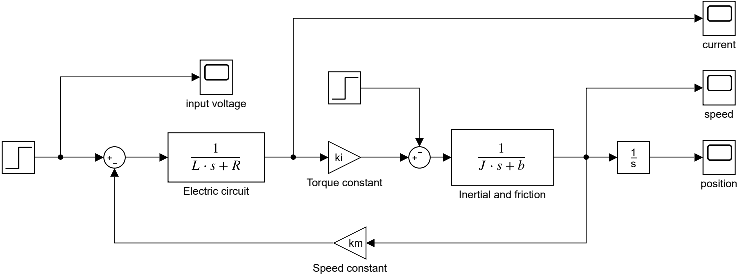
Exercise 61 - continued





Exercise 62

Implement a Simulink model of a DC motor as shown.

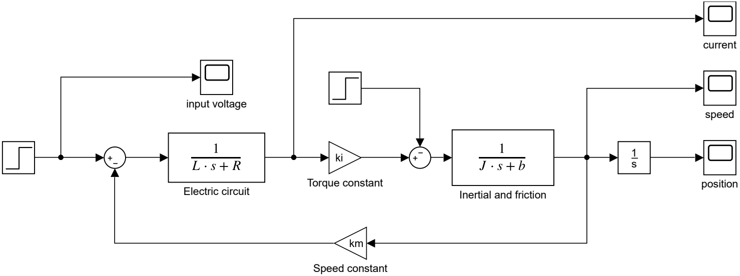


Consider the following motor characteristics: V = 12 V, R = 2 Ω, L = 0.01 H, $K_a = 0.2$, $J = 0.000222\ Kg.m^2$, b = 0.002 Nm/sec.

Evaluate the influence of viscous friction, inertia, and K_a on the steady-state speed.

Exercise 63

Implement a proportional **speed controller** for motor model from exercise 62.

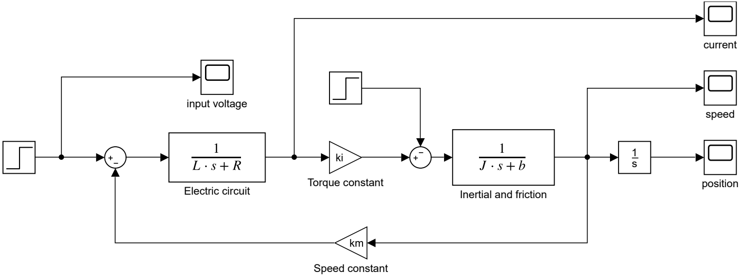


Consider the following motor characteristics: V = 12 V, R = 2 Ω, L = 0.01 H, $K_a = 0.2$, $J = 0.000222\ Kg.m^2$, b = 0.002 Nm/sec.

Evaluate the influence of viscous friction, inertia, and K_a on the transient response.

Exercise 64

Implement a proportional **position controller** for the motor model from exercise 62.

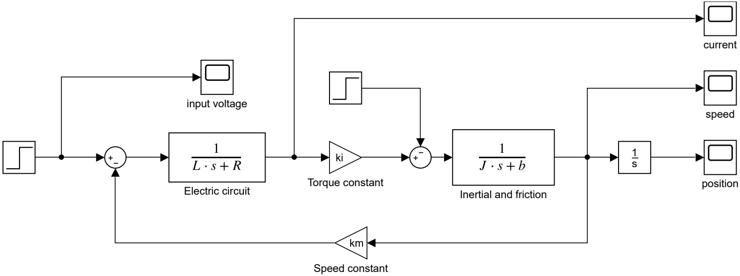


Consider the following motor characteristics: V = 12 V, R = 2 Ω, L = 0.01 H, $K_a = 0.2$, $J = 0.000222\ Kg.m^2$, b = 0.002 Nm/sec.

Evaluate the influence of viscous friction, inertia, and K_a on the transient response.

Exercise 65

Implement a proportional **torque controller** for the motor model from exercise 62.



Consider the following motor characteristics: V = 12 V, R = 2 Ω, L = 0.01 H, $K_a = 0.2$, $J = 0.000222\ Kg.m^2$, b = 0.002 Nm/sec.

Evaluate the influence of viscous friction, inertia, and K_a on the transient response.

Next class...

- Synchronous motors

Additional supporting materials for Lecture 13:

DC motor working principle: `https://youtu.be/LAtPHANEfQo`

DC generators: `https://youtu.be/0pL0joqJmqY`

DC motor modelling: `https://bit.ly/1l3rajk`