METE 3100U
Actuators and Power Electronics

Lecture 10

Mechanical Force in Electromagnetic Systems

Outline of Lecture 10

By the end of today's lecture, you should be able to

- Establish the relationship to express electromagnetic torque and force
- Calculate the force developed by an electromagnet
- Understand the principle of rotary and linear machines

Applications

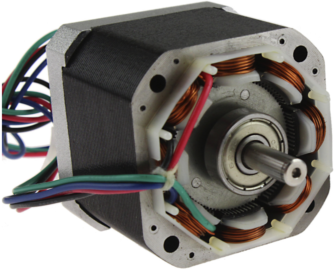
Electromagnets are widely used in industrial settings to lift heavy loads. What is the maximum weight it can lift?

Applications

Security systems use locking systems for doors which are generally magnetic locking systems. What is the required force to unlock the door?

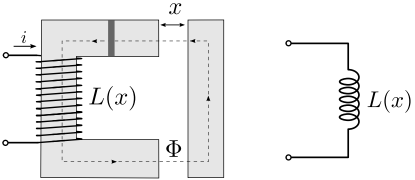
Applications

Stepper motors are widely used in mechatronic and robotic applications. What is the relation between the rotor's position and the applied torque?



Energy conversion process

A coil may be represented by an ideal inductance



The flux linkage λ and inductance L are

$$\lambda = N\phi, \qquad L = \frac{\lambda}{i} \tag{1}$$

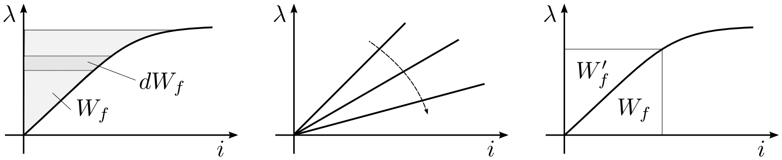
Thus

$$L = \frac{N\phi}{i} = \frac{NBA}{i} = \frac{N\mu HA}{i} \tag{2}$$

Recall that $H\ell(x) = Ni$, thus

$$L(x) = \frac{N\mu HA}{\frac{H\ell(x)}{N}} = \frac{N^2}{\frac{\ell(x)}{\mu A}} = \frac{N^2}{\mathcal{R}(x)} \tag{3}$$

Energy and coenergy (from lecture 9)



$$W_f = \int_0^\lambda i d\lambda, \qquad W_f = \int_0^i \lambda di \tag{4}$$

For a linear system

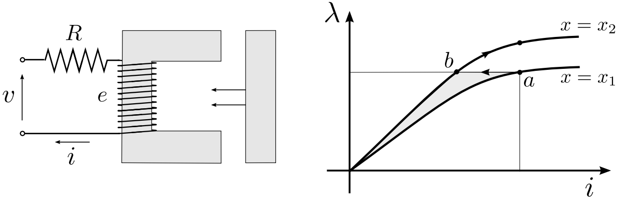
$$W'_f = W_f \tag{5}$$

→ W_f if the field energy

→ W'_f if the field coenergy

Mechanical force (from lecture 9)

Case 1: The movement occurs quickly. What happens to the current?



If λ is constant, the work done is a decrease in the field energy

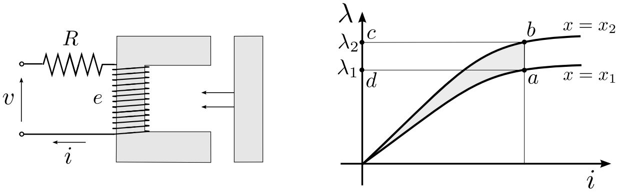
$$dW_m = dW_f \tag{6}$$

If f_m is the mechanical force causing a displacement dx

$$f_m dx = dW_f$$
$$f_m = \left.\frac{\partial W_f(\lambda, x)}{\partial x}\right|_{\lambda=cte}$$

Mechanical force (from lecture 9)

Case 2: The movement occurs slowly. What happens to the current?



If i is constant, the work done is an decrease in the field co-energy

$$dW_m = dW_f' \tag{7}$$

If f_m is the mechanical force causing a displacement dx

$$f_m dx = dW_f'$$

$$f_m = \left.\frac{\partial W_f'(i,x)}{\partial x}\right|_{i=cte}$$

Mechanical force

How can the force be related to the air gap?

$$\lambda = L(x)i \quad (8)$$

Note that $L(x)$ depends on the air gap length

$$W_f = \int i d\lambda = \int_0^{\lambda} \frac{\lambda}{L(x)} d\lambda$$

$$W_f = \frac{\lambda^2}{2L(x)} = \frac{[L(x)i]^2}{2L(x)} = \frac{1}{2}L(x)i^2$$

For a linear system

$$W_f = W_f' = \frac{1}{2}L(x)i^2 \quad (9)$$

Energy conversion process

Since

$$f_m = \left.\frac{\partial W_f(\lambda, x)}{\partial x}\right|_{\lambda=cte} = \left.\frac{\partial W_f'(i, x)}{\partial x}\right|_{i=cte}$$

Using the energy formulation

$$f_m = \left.\frac{\partial}{\partial x}\left(\frac{\lambda^2}{2L(x)}\right)\right|_{\lambda=cte}$$

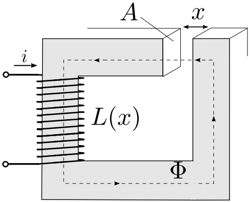
$$f_m = \frac{\lambda^2}{2L(x)^2}\frac{dL(x)}{dx} = \frac{1}{2}i^2\frac{dL(x)}{dx}$$

Using the co-energy formulation

$$f_m = \left.\frac{\partial}{\partial x}\left(\frac{1}{2}L(x)i^2\right)\right|_{i=cte}$$

$$f_m = \frac{\lambda^2}{2L(x)^2}\frac{dL(x)}{dx} = \frac{1}{2}i^2\frac{dL(x)}{dx}$$

Mechanical force



$$f_m = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$
$$B = \frac{\mu N i}{\ell}$$
$$\mathcal{R} = \frac{\ell}{\mu A}$$

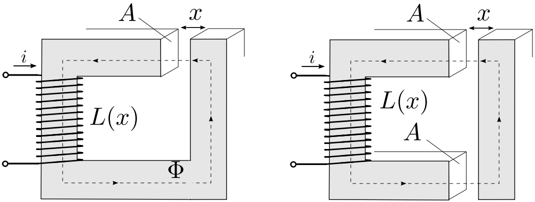
For the system above and neglecting the core reluctance:

$$f_m = \frac{1}{2} i^2 \frac{d}{dx}\left(\frac{N^2}{\mathcal{R}(x)}\right) = \frac{1}{2} i^2 \frac{d}{dx}\left(\frac{N^2}{\frac{x}{\mu A}}\right) \tag{10}$$

$$f_m = \frac{1}{2} i^2 N^2 \mu A \frac{d}{dx}\left(\frac{1}{x}\right) = \frac{1}{2} \mu A \left(\frac{iN}{x}\right)^2 \tag{11}$$

$$f_m = \frac{1}{2} \mu A \left(\frac{B}{\mu}\right)^2 = \frac{B^2}{2\mu} A \tag{12}$$

Mechanical force



The force between both sides of the air gap is

$$f_m = \frac{B^2}{2\mu} A \tag{13}$$

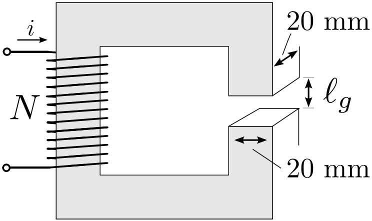
A is the area of the gap. Thus, the force per unit area of air gap

$$F_m = \frac{B^2}{2\mu} \tag{14}$$

is called the magnetic pressure (N/m^2).

Exercise 42

The magnetic circuit shown and a coil with 500 turns. The length of the air gap is 10 mm. If a current $i = 2$ A runs in the coil, calculate

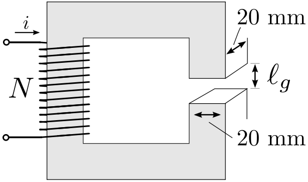


(a) The attraction force between both sides of the air gap

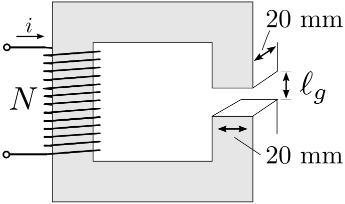
(b) The energy stored in the air gap

Neglect the reluctance of the core and the leakage flux.

Exercise 42 - continued

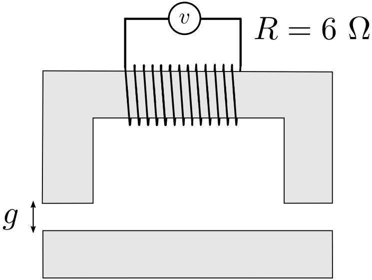


Exercise 42 - continued



Exercise 43

The electromagnet has a square cross section of 6 × 6 cm^2. The coil has 300 turns and a resistance of 6 Ω. The reluctance of the magnetic core and flux leakages are negligible.

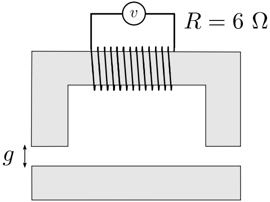


If the air gap is held at 5 mm and $v = 120$ V DC, determine

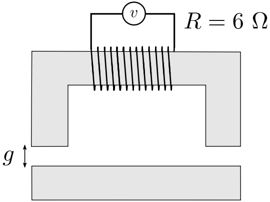
(a) The stored field energy

(b) The lifting force

Exercise 43 - continued

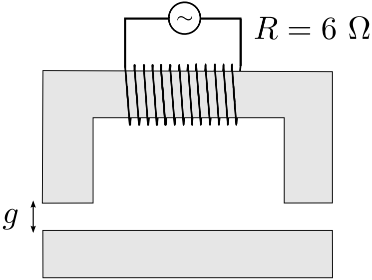


Exercise 43 - continued



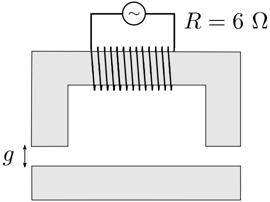
Exercise 44

The electromagnet has a square cross section of 6 × 6 cm^2. The coil has 300 turns and a resistance of 6 Ω. The reluctance of the magnetic core and flux leakages are negligible.

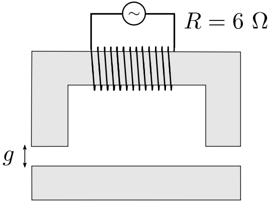


If the air gap is held at 5 mm and an AC source of 120 V at 60 Hz is connected to coil, determine the average value of the lifting force.

Exercise 44 - continued

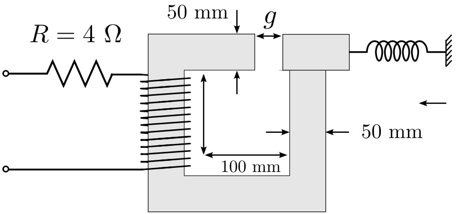


Exercise 44 - continued



Exercise 45

The actuator shown is made of cast iron with a relative permeability of 1136 At/Wb. The coil has 500 turns and a resistance of 4 Ω. The air gap is $g = 1$ mm. All dimensions are in millimetres.



(a) What is the coil voltage required to stablish an air gap flux density of 0.5 T.

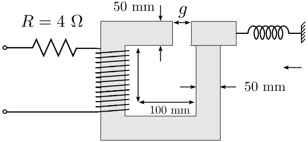
(b) What is the stored energy in the actuator?

(c) What is the attraction force of the actuator arm?

(d) What is the inductance of the coil?

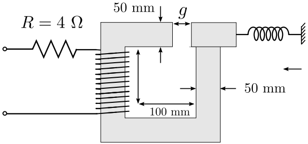
Exercise 45 - continued

(a) The coil voltage required to stablish an air gap flux density of 0.5 T.



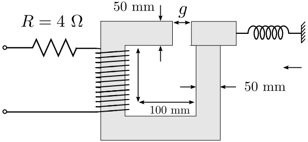
Exercise 45 - continued

(b) The stored energy in the actuator.



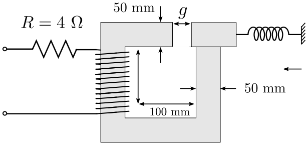
Exercise 45 - continued

(c) The attraction force of the actuator arm.



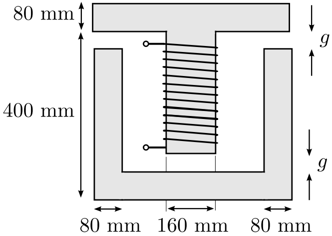
Exercise 45 - continued

(d) The inductance of the coil.



Exercise 46

The figure shows an electromagnet used to lift a section of steel channel. The coil has 600 turns and the reluctance of the magnetic material can be neglected up to a flux density of 1.4 T.



If the current in the coil is 15 A, determine

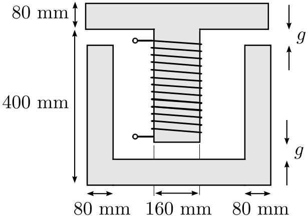
(a) The maximum air gap g for which $B = 1.4$ T.

(b) The force on the steel channel.

(c) If the steel channel has a mass of 1000 kg, what is the largest air gap at which the steel channel can be lifted?

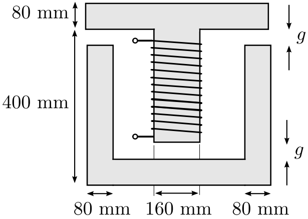
Exercise 46 - continued

(a) The maximum air gap g for which $B = 1.4$ T.



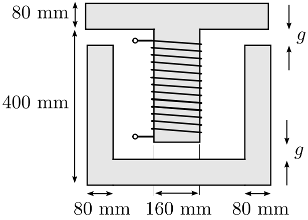
Exercise 46 - continued

(b) The force on the steel channel.



Exercise 46 - continued

(c) If the steel channel has a mass of 1000 kg, what is the largest air gap at which the steel channel can be lifted?



Next class...

- Rotating machines

Additional fun/useless stuff related to Lecture 10:

Adafruit electromagnet: `https://www.adafruit.com/product/3875`

Solenoid engine V1: `https://youtu.be/xc14jBSlD64`

Solenoid engine V8: `https://youtu.be/2wfo2QqkXMU`