MECE 2430U
Dynamics

Lecture 17

Relative Motion Analysis: Acceleration

Outline of Lecture 17

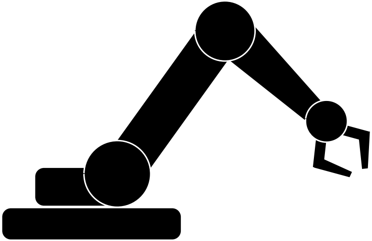
By the end of this lecture you should be able to

- Resolve the acceleration of a point into translation and rotation
- Determine acceleration of a point using a relative acceleration analysis

Applications

A collaborative robot is intended to physically interact with humans in a shared workspace.

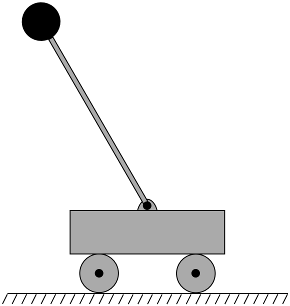
To ensure safely, the acceleration of the arm must be limited. How can it be calculated?



Applications

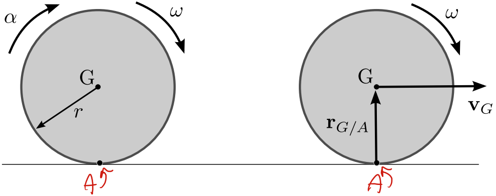
An inverted pendulum has its center of mass above its pivot point. It is unstable and without additional help will fall over.

How can we calculate the acceleration of the pendulum?



Instantaneous centre of zero velocity

There always exists a point in the plane of motion of body where the velocity is zero.



$$\mathbf{v}_G = \mathbf{v}_A + \omega \times \mathbf{r}_{G/A}$$

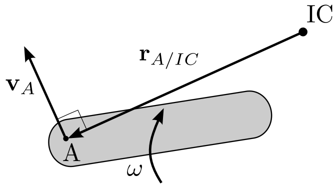
$$= \mathbf{0} + (-\omega)\mathbf{k} \times (r\mathbf{j}) = r\omega \vec{i}$$

→ A is called the centre of zero velocity, or IC

→ IC may not lie on the body

Instantaneous centre of zero velocity

Case 1: The velocity of a point A and the angular velocity ω are known:

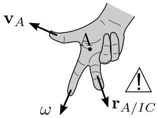


$$\mathbf{v}_A = \mathbf{v}_{IC} + \omega \times \mathbf{r}_{A/IC}$$

0

→ IC lies along a line perpendicular to $\mathbf{v}_A$.

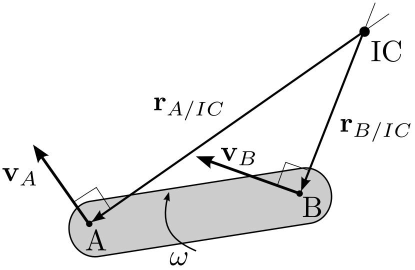
$$r_{A/IC} = \frac{v_A}{\omega}$$



→ Note that the body rotates CW

Instantaneous centre of zero velocity

Case 2: The line of action of two non-parallel velocities are known:

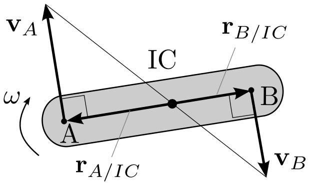


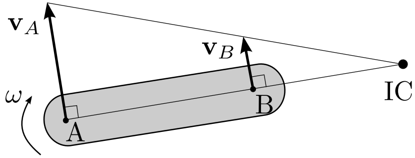
→ *IC* lies along a the intersection of the lines perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

$$r_{A/IC} = \frac{v_A}{\omega}, \qquad r_{B/IC} = \frac{v_B}{\omega}$$

Instantaneous centre of zero velocity

Case 3: The line of action and **magnitude** of two parallel velocities are known:





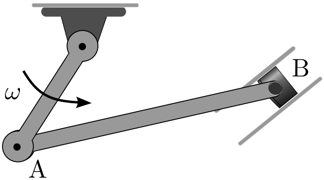
→ IC lies along a the intersection of the lines perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

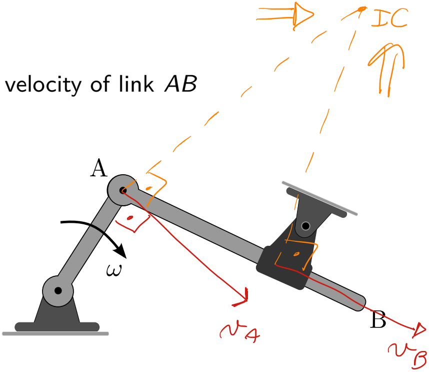
$$r_{A/IC} = \frac{v_A}{\omega}, \qquad r_{B/IC} = \frac{v_B}{\omega}$$

Experiment

Exercise 1

Locate the instantaneous centre of zero velocity of link AB





Relative motion analysis - acceleration

The equation relating the velocity of two points is

$$\frac{d\mathbf{r}_B}{dt} = \frac{d\mathbf{r}_A}{dt} + \frac{d\mathbf{r}_{B/A}}{dt}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

For the acceleration, we have

$$\frac{d\mathbf{v}_B}{dt} = \frac{d\mathbf{r}\mathbf{v}_A}{dt} + \frac{d\mathbf{v}_{B/A}}{dt} \quad \Rightarrow \quad \vec{a_B} = \vec{a_A} + \vec{a_{B/A}}$$

$\frac{d\mathbf{v}_A}{dt} = \mathbf{a}_A$: Absolute acceleration of A

$\frac{d\mathbf{v}_B}{dt} = \mathbf{a}_B$: Absolute acceleration of B

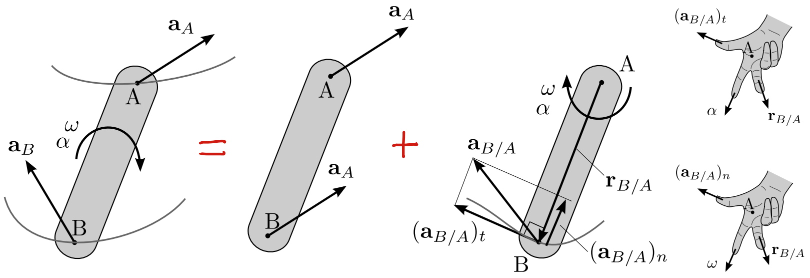
$\frac{d\mathbf{v}_B}{dt} = \mathbf{a}_B$: Acceleration of B with respect to A

Relative motion analysis - acceleration

In terms of tangential and normal components:

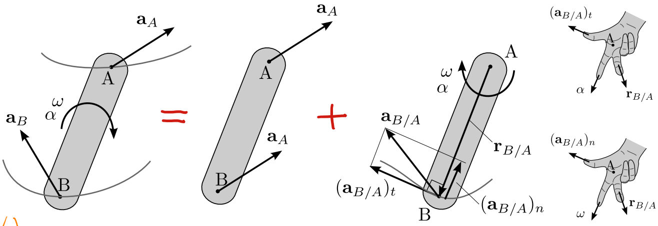
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$= \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$



Recall that $a_t = \alpha \times \mathbf{r}$ and $a_n = -\omega^2\mathbf{r}$

Relative motion analysis - acceleration



α is a vector

$$\mathbf{a}_B = \mathbf{a}_A + \vec{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

ω is a scalar

Due to rotation

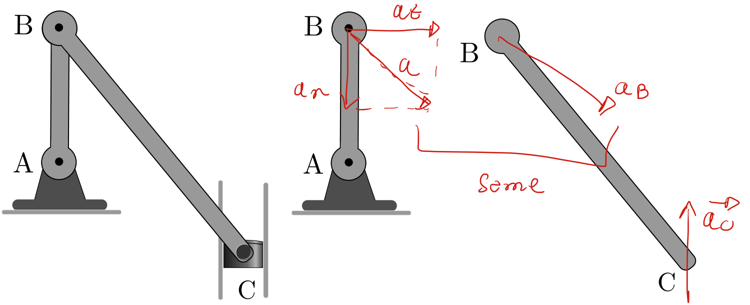
Due to translation

- → $\mathbf{a}_B$: acceleration of point B
- → $\mathbf{a}_A$: acceleration of point A
- → α: angular acceleration of the body
- → ω: angular velocity of the body
- → $\mathbf{r}_{B/A}$: position vector from A to B

Pin connections

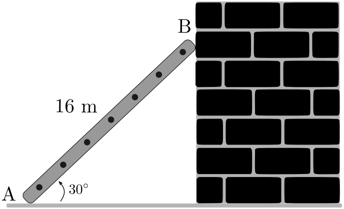
Pin-connected points experience the same acceleration as they travel along the same path

The acceleration of C is directed vertically. Why?



Exercise 2

At a given instant the bottom A of the ladder has an acceleration $a_A = 4\ \text{m/s}^2$ and velocity $v_A = 6\ \text{m/s}$ both acting to the left. Determine the acceleration of the top of the ladder and the ladder's angular acceleration at this same instant.



Procedure:

→ Find CI and determine ω

→ Apply the relative acceleration equation to find a_B and α

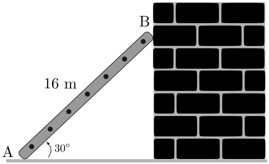
Exercise 2 - continued

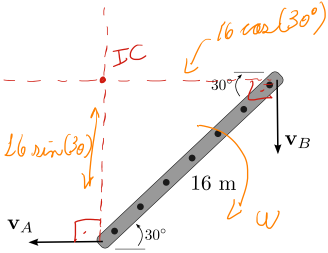
→ Angular velocity

$v_A = \omega \, r_{A/IC}$

$6 = 8\omega$

$\boxed{\omega = 0.75 \text{ rad/s}}$





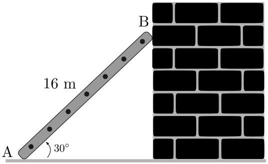
Exercise 2 - continued

→ Angular acceleration

⚠ ω is a scalar. $= 0.75$ rad/s

$$\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{B/A} - \omega^2 \vec{r}_{B/A}$$

$$\mathbf{r}_{B/A} = 16(\cos 30^\circ \mathbf{i} + \sin 30^\circ \mathbf{j})\ \text{m}$$



$$-a_B \vec{j} = -4\vec{i} + (\alpha \vec{k}) \times (16 \cos 30 \vec{i} + 16 \sin(30) \vec{j}) - 0.75^2 (16 \cos \vec{i} + 16 \sin \vec{j})$$

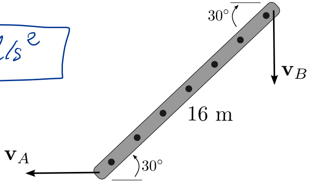
$$-a_B \vec{j} = -4\vec{i} + 13.856\alpha \vec{j} - 8\alpha \vec{i} - 7.79 \vec{i} - 4.5 \vec{j}$$

(i) $0 = -4 - 8\alpha - 7.79$

$$\boxed{\alpha = 1.47\ \text{rad/s}^2}$$

(j) $-a_B = 13.856\alpha - 4.5$

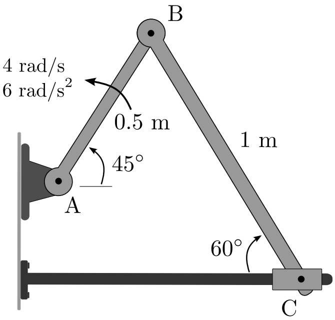
$$\boxed{a_B = 24.9\ \text{m/s}^2}$$



↻ $\alpha > 0$ → what does it mean?

Exercise 3

Bar AB has the angular velocity and acceleration shown. Determine the velocity and acceleration of the slider block C at this instant.



Procedure:

→ Calculate $\mathbf{v}_B$ and $\mathbf{a}_B$ considering rotation about A

→ Find IC and determine v_B and v_C

→ Find $\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2 \mathbf{r}_{C/B}$

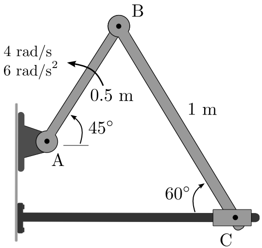
Exercise 3 - continued

1 → Rotation about A

$\vec{v}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$\vec{v}_B = 4\vec{k} \times \left(0.5\cos(45)\vec{i} + 0.5\sin(45)\vec{j}\right)$

$\vec{v}_B = -\sqrt{2}\vec{i} + \sqrt{2}\vec{j}$, $v_B = 2\ m/s$

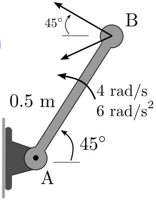


Acceleration $\mathbf{a}_B$

$\mathbf{a}_B = \vec{\alpha}_{AB} \times \mathbf{r}_{AB} - \omega_{AB}^2 \mathbf{r}_{AB}$ (or B/A, or B/A)

$\vec{a}_B = 6\vec{k} \times (0.5\cos 45\vec{i} + 0.5\sin(45)) - 4^2(0.5\cos 45\vec{i} + 0.5\sin(45)\vec{j})$

$$\boxed{\vec{a}_B = -5.5\sqrt{2}\,\vec{i} - 2.5\sqrt{2}\,\vec{j}\ \ m/s^2}$$



Exercise 3 - continued

2 → Calculate v_B and v_C

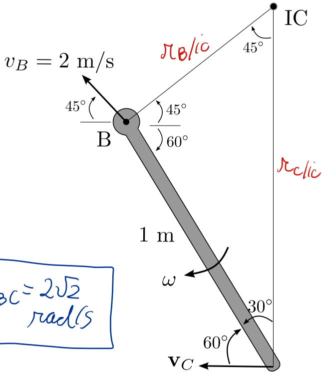
$$\frac{r_{B/IC}}{\sin(30^\circ)} = \frac{1}{\sin(45^\circ)} \rightarrow r_{B/IC} = \frac{\sqrt{2}}{2}\ m$$

$$\frac{r_{C/IC}}{\sin(45+60)} = \frac{1}{\sin(45)} \rightarrow r_{C/IC} = 1.366\ m$$

$$v_B = \omega_{BC}\, r_{B/IC} \qquad 2 = \omega_{BC}\left(\frac{\sqrt{2}}{2}\right) \qquad \boxed{\omega_{BC} = 2\sqrt{2}\ rad/s}$$

$$v_C = \omega_{BC}\, r_{C/IC}$$

$$\boxed{v_C = 3.86\ m/s}$$



Exercise 3 - continued

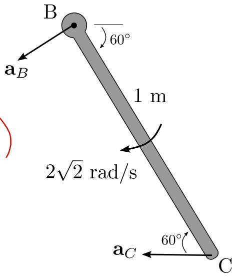
3 → Relative acceleration equation

$\mathbf{a}_B = -5.5\sqrt{2}\mathbf{i} - 2.5\sqrt{2}\mathbf{j}$ m/s^2

$\mathbf{r}_{C/B} = 1\cos 60^\circ\mathbf{i} - 1\sin 60^\circ\mathbf{j}$ m

$$\mathbf{a}_C = \mathbf{a}_B + \vec{\alpha}_{BC} \times \mathbf{r}_{C/B} - \omega_{BC}^2\mathbf{r}_{C/B}$$

C w.r. to B (how to get to C from B)



$$-a_C\vec{i} = -5.5\sqrt{2}\vec{i} - 2.5\sqrt{2}\vec{j} - \alpha_{BC}k \times (\cos 60\,\vec{i} - \sin(60)\vec{j}) - (2\sqrt{2})^2(\cos 60\,\vec{i} - \sin 60\,\vec{j})$$

$$-a_C\vec{i} = \left(-\frac{\sqrt{3}}{2}\alpha_{BC} - 11.77\right)\vec{i} + (3.392 - 0.5\alpha_{BC})\vec{j}$$

(j) $0 = 3.392 - 0.5\alpha_{BC}$

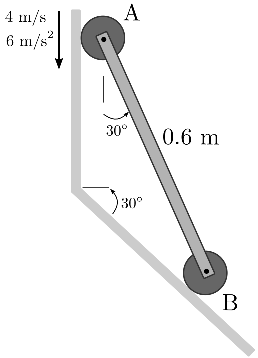
$\alpha_{BC} = 6.785$ rad/s^2 ↻

(i) $-a_C = -\frac{\sqrt{3}}{2}(6.785) - 11.77$

$a_C = 17.7$ m/s ←

Exercise 4

At a given instant the roller A on the bar has the velocity and acceleration shown. Determine the velocity and acceleration of the roller B, and the bar's angular velocity and angular acceleration at this instant.



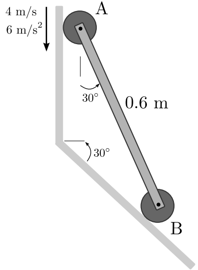
Procedure:

→ Find IC and determine ω and v_B

→ Apply the relative acceleration equation

Exercise 4 - continued

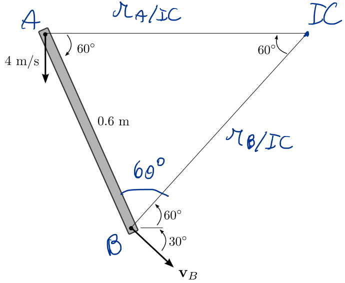
1 → Locate IC and find ω and v_B



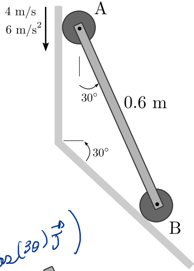
from the geometry $r_{A/IC} = r_{B/IC} = 0.6\,m$

$v_A = \omega\, r_{A/IC}$ $\quad \omega = \dfrac{4}{0.6} = 6.67\ rad/s$

$v_B = \omega\, r_{B/IC} = 6.67(0.6) = 4\,m/s$ ↘



Exercise 4 - continued



2 → Relative acceleration equation

$\mathbf{a}_B = a_B(\cos 30^\circ \mathbf{i} - \sin 30^\circ \mathbf{j})$

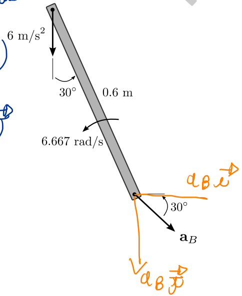
$\mathbf{r}_{B/A} = 0.6(\sin 60^\circ \mathbf{i} - \cos 30^\circ \mathbf{j})$

$\mathbf{a}_B = \mathbf{a}_A + \vec{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$

$$a_B(\cos 30 \vec{i} - \sin 30 \vec{j}) = -6\vec{j} + \alpha \vec{k} \times 0.6(\sin 30 \vec{i} + \cos(30)\vec{j}) - 0.667^2 \cdot 0.6(\sin(30)\vec{i} + \cos 30 \vec{j})$$

$$\frac{\sqrt{3}}{2} a_B \vec{i} - \frac{1}{2} a_B \vec{j} = (0.3\sqrt{3}\alpha - 13.33)\vec{i} + (0.3\alpha + 17.09)\vec{j}$$

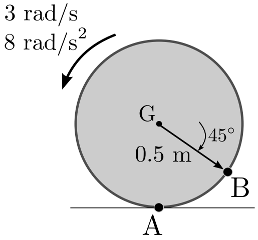
$$\alpha = -15.66 \text{ rad/s}^2$$
$$a_B = 24.8 \text{ m/s}^2$$



Exercise 5

The disk has an angular acceleration $\alpha = 8$ rad/s^2 and angular velocity $v = 3$ rad/s at the instant shown. If it does not slip at A, determine the acceleration of point B.

Try this at home.

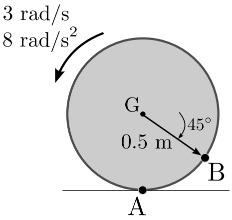


Procedure:

→ Apply the relative acceleration equation

→ Calculate the magnitude and direction of **a**

Exercise 5 - continued



Hint: Since the disk rolls without slipping $a_O = \alpha r$

Relative acceleration equation

$$\vec{a_B} = \vec{a_O} + \vec{\alpha} \times \mathbf{r}_{B/O} - \omega^2 \mathbf{r}_{B/O}$$

$$\vec{a_B} = -4.354\vec{i} + 6.010\vec{j} \quad m/s^2$$

Quiz

Q1 → Consider the following statements:

Statement 1: *IC* has always zero acceleration.

Statement 2: The location of *IC* does not change over time.

Statements 1 and 2 are, respectively:

(a) True and true

(b) False and true

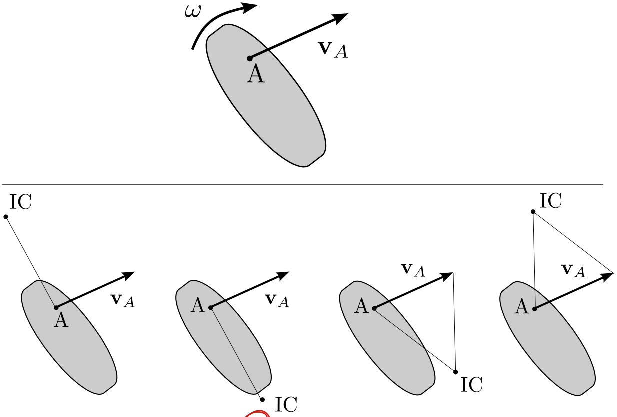
(c) True and false

(d) False and false

(e) None of the above

Quiz

Q2 → Select the correct location of *IC* for the body shown



(a) (b) (c) (d)

Why not a?

Quiz

Q3 → If points P and P' are in contact with one another without slipping, at the instant shown they have:

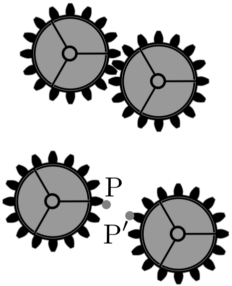
(a) The same position

(b) The same velocity

(c) The same speed

(d) The same acceleration

(e) All of the above



Quiz

Q4 → If points A and A' are pin-connected, at the instant shown they have:

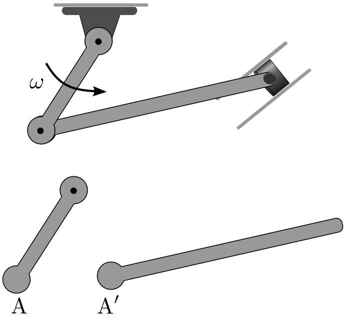
(a) The same position

(b) The same velocity

(c) The same speed

(d) The same acceleration

(e) All of the above



Next class...

- Mass moment of inertia