

Penetration Depth Quantification of Open-ended Coaxial Probes for Dielectric Spectroscopy of Layered Media

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Abstract—Dielectric spectroscopy using open-ended coaxial probes is a powerful tool for biological tissue classification. It measures the complex permittivity of a medium as a function of frequency by applying an electromagnetic field and observing the energy reflected back. In heterogeneous tissue, a critical parameter that defines the accuracy of permittivity measurement is the penetration depth (PD) of the electromagnetic field for a given probe geometry, however, it is still unclear how the tissue characteristics affect the PD and the accuracy of the measurements. This paper evaluates the effect of various tissue and probe parameters on the PD in the context of dielectric spectroscopy through an open-ended coaxial probe. The PD is evaluated under different simulation conditions considering a probe inserted into a 2-layered tissue with different dielectric characteristics in 54,000 different simulations. A model for extracting the permittivity from the simulated reflection coefficient is also described. The results show for the first time that the PD increases as the difference in permittivity and conductivity between the layers increases, suggesting that measurement accuracy is sensitive to changes of contrast in the layer's characteristics. The results also show that the PD decreases with the excitation frequency but increases with the diameter of the coaxial probe. These findings can greatly aid in quantifying and understanding the sensitivity of biological tissue classification using dielectric spectroscopy.

Index Terms—Dielectric spectroscopy; penetration depth; tissue characterization; coaxial probe.

I. INTRODUCTION

Dielectric spectroscopy via open-ended coaxial probes is a powerful biosensing tool for tissue characterization [1], [2]. It exploits the interfacial polarization across cell membranes induced by an external electromagnetic field that sweeps over a wide frequency range. This polarization represents the

This research is supported by the Canadian Cancer Society and the Canadian Institutes of Health Research (grant #944486). Cette recherche est appuyée par la Société canadienne du cancer et les instituts de recherche en santé du Canada (subvention no 944486).

medium's permittivity, or its resistance to the establishment of the electric field. In simpler terms, permittivity denotes the medium's capacity to store energy in the form of an electric field. By analyzing the electromagnetic energy the tissue reflects back to the probe, the average permittivity of the polarized region within the tissue can be extracted.

Open-ended coaxial probes are mostly used for classifying heterogeneous and layered media, e.g., for cancer detection. A parameter that plays an important role in the classification's accuracy is the penetration depth (PD) of the electromagnetic field in the tissue [3], [4]. The PD can be defined as the medium thickness ahead of the probe's tip that significantly affect the reflected electromagnetic field and the inferred permittivity. For example, Hagl *et al.* report that the depth at which breast tissue characterization is accurate cannot exceed the diameter of the probe [5]. This observation has strong implications for approaches that neglect the effect of a middle layer between the probe's tip and the target tissue. For instance, *in-vivo* breast cancer measurements have been conducted with a small diameter probe in contact with the skin [6]. Classification is only accurate if the tissue under test is slightly beyond the skin layer. A probe with an outer diameter of 2.2 mm was used in [7] for *in-vivo* measurement of skin cancer located 0.15 mm below the stratum corneum.

Different definitions for the PD have been proposed. Meaney *et al.* consider a coaxial probe inserted in a two-layer tissue. The PD is defined as the distance between the probe's tip and the second layer where the measured permittivity drops 20% below the expected value from a linear extrapolation of the permittivity as a function of the distance [8]. This definition is only applicable when the measured permittivity can be given as a weighted sum of each layer's permittivity, and the weights are linearly dependent on the distance of the probe to the

second layer. Aydinalp *et al.* defined the PD as the distance from the second layer where the measured permittivity changes by 5% compared to the first layer's permittivity [9]. Their simulation results show that the PD is frequency dependent; however, this could not be validated experimentally due to noise. These different definitions highlight the fact that, in a multilayered tissue, whenever the separation layer is within the PD of the probe, the observed measurement is a mixture of the signature of two unknown materials and therefore does not convey specific information. In medical applications, a probe with a small penetration depth is therefore more beneficial.

A question that has not been addressed in detail is the effect of the permittivity contrast of the tissue layers on the PD and, consequently, the influence of the PD on the permittivity measurement itself. To answer this question, this paper proposes a definition for the penetration depth that is based on the change in the measured permittivity as the probe is inserted from one layer toward the other. It should be noted that the region preceding the probe tip does not affect the measurement. Thus, the scenario can be likened to situations involving either the placement of the probe on top of a thin layer (e.g., skin) or its insertion into a two-layer material (e.g., a tumour within healthy tissue).

In addition, an efficient method for extracting the permittivity from the measured reflection coefficient is presented, which is used to determine the PD according to the proposed definition. The PD is then quantified for different tissue layer contrasts, probe diameters, and frequencies using a total of 54,000 simulations performed in Ansys. The simulation results show that the permittivity measurement accuracy is dependent on the layer's permittivity and conductivity contrasts and that the PD increases with the contrast. Furthermore, it is observed that the PD increases with the probe diameter. Finally, the simulation results shed light on the contradictory findings previously reported in the literature regarding the effect of frequency on the PD. It is shown that in fact, the PD decreases with frequency.

To the best of the author's knowledge, such comprehensive simulation and evaluation of the PD have not been presented before. The observations highlighted in this paper can aid in determining the sensitivity and accuracy of the permittivity measurement of layered tissue, which is a crucial factor affecting the performance of tissue identification using dielectric spectroscopy.

II. PENETRATION DEPTH IN LAYERED MEDIA

In dielectric spectroscopy, an electromagnetic wave is radiated through a coaxial probe to the material under test (MUT). The wave is partially reflected back to the probe at a rate that depends on the dielectric properties of the material. The ratio of the reflected and transmitted complex amplitude of the wave is called the reflection coefficient, from which the permittivity is extracted. Consider the two-layer tissue shown in Fig. 1(a). The probe is inserted in the first layer, whose relative permittivity is ϵ_{r1} , at distance ℓ_1 from the second layer, whose permittivity is ϵ_{r2} . So long as $\ell_1 > 0$, the expected

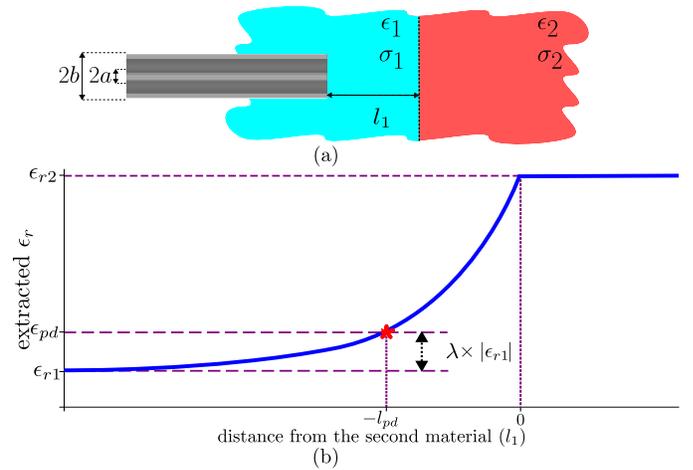


Fig. 1. (a) Coaxial probe inside a two-layer material having a complex relative permittivity $\epsilon_m = \epsilon_r - j \frac{\sigma}{\omega \epsilon_0}$, where ϵ_r is the relative permittivity, σ is the conductivity, ω is the angular frequency, and ϵ_0 is the vacuum's permittivity. (b) Observed permittivity as a function of the probe's distance to the second layer used to define the PD, here shown as ℓ_{pd} .

measured relative permittivity is ϵ_{r1} . However, the second layer will also affect the measurement and the calculated relative permittivity is a weighted average of ϵ_{r1} and ϵ_{r2} . The PD is defined as:

$$PD = \ell_1 \quad \text{when} \quad |\epsilon_m - \epsilon_{r1}| = \lambda |\epsilon_{r1}| \quad (1)$$

where ϵ_m is the measured relative permittivity, and λ is a constant such that $0 < \lambda < 1$, see Fig. 1(b). This definition entails that the PD is the minimal distance from the tip of the probe to the second tissue layer, after which the properties of the second layer significantly affect the measured permittivity, i.e., the difference between the measured and expected permittivity $|\epsilon_m - \epsilon_{r1}|$ is $\lambda \times 100\%$ greater than ϵ_{r1} , if $\epsilon_{r2} > \epsilon_{r1}$, or $\lambda \times 100\%$ smaller than ϵ_{r1} if $\epsilon_{r2} < \epsilon_{r1}$. The advantage of this definition compared to others is that it only relies on the expected permittivity value of the material in contact with the probe's aperture.

A. Permittivity Value Extraction

The next step for determining the PD is to extract the permittivity from the reflection coefficient; a parameter that is typically measured using a vector network analyzer (VNA). The relation between the normalized aperture admittance Y of the probe and the measured reflection coefficient Γ is:

$$Y = \frac{1 - \Gamma}{1 + \Gamma} \quad (2)$$

The normalized aperture admittance of the probe given a permittivity value and frequency can be calculated using a Taylor expansion as [10]:

$$Y_s = \frac{jk_m^2}{2\pi k_c} \sum_{n=1}^{\infty} \frac{(jk_m)^n}{n!} I_n \quad (3)$$

TABLE I
PROBE COEFFICIENTS

Probe Type	I_0	I_1	I_2	I_3	I_4
Thin	7.38×10^{-4}	-8.74×10^{-9}	-1.30×10^{-11}	1.08×10^{-15}	3.58×10^{-18}
Thick	4.58×10^{-3}	-4.45×10^{-7}	-3.65×10^{-9}	9.48×10^{-12}	-1.45×10^{-14}

where $k_m = \omega\sqrt{\epsilon_r\epsilon_0\mu_0}$ and $k_c = \omega\sqrt{\epsilon_c\epsilon_0\mu_0}$ are the MUT and probe dielectric complex-value wave-number, respectively. The coefficients I_n are determined via model fitting using simulation results of materials with different permittivity values at different frequencies. Let vector b be the normalized aperture admittance of the simulated material as follow:

$$b = [Y_{sim,1} \quad Y_{sim,2} \quad \cdots \quad Y_{sim,i}]^T \quad (4)$$

and A be the simulated material wavenumbers matrix of the model as follow:

$$A = \frac{1}{2\pi k_c} \begin{bmatrix} k_{m,1}^2 & jk_{m,1}^3 & \cdots & jk_{m,1}^{n+2} \\ k_{m,2}^2 & jk_{m,2}^3 & \cdots & jk_{m,2}^{n+2} \\ \vdots & \vdots & \ddots & \vdots \\ k_{m,p}^2 & jk_{m,p}^3 & \cdots & jk_{m,p}^{n+2} \end{bmatrix}. \quad (5)$$

The vector containing the probe's coefficients is defined as:

$$x = [I_0 \quad I_1 \quad \cdots \quad I_n]^T \quad (6)$$

All the values in 4 and 5 can be obtained from the simulations. The coefficients I_n in 6 are determined by minimizing $\|b - Ax\|_2$ using the least squares method.

For $n = 4$, the coefficients I_n for two standard 50- Ω Teflon-filled coaxial cables with dielectric radii of $b = 0.43$ mm and $b = 2.6$ mm, and inner conductor radii of $a = 0.14$ mm and 0.82 mm, respectively are given in Table I. These probes will hereafter be referred to as thin and thick probes, respectively. Having the coefficients I_n , the permittivity can be extracted from the measured reflection coefficient. While the proposed model in 3 gives the normalized aperture admittance based on material wavenumber k_m , the roots of this equation when it is equated to the measured admittance is the k_m value at the measured frequency. The complex relative permittivity (ϵ_r) then is:

$$\epsilon_r = \frac{k_m^2}{\omega^2\epsilon_0\mu_0} \quad (7)$$

The solution of the above is the value that satisfies the physical constraint ($\epsilon_r > 1$ and $\sigma > 0$).

III. PENETRATION DEPTH QUANTIFICATION

Fig. 2 shows the simulation setup in ANSYS HFSS, which is composed of two cubic media with different relative permittivities. Radiation boundaries are assigned to the outer walls to prevent reflection and emulate unbounded boundaries. The permittivity and conductivity ranges of the two media for all simulations are selected based on measured data from biological tissue [11]. The probe's tip is initially positioned in the first material at a distance of $l_1 = -0.7$ mm from the second layer. Then, it is gradually moved closer to the second material

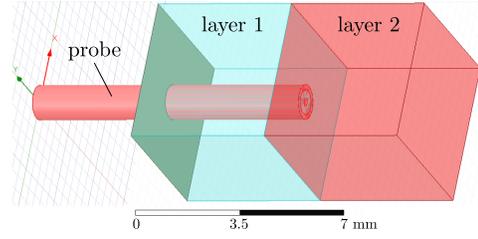


Fig. 2. Full-wave simulation of an open-ended coaxial probe in the MUT with Ansys. The blue and red cubes are the first and second materials, respectively.

in steps of 0.01 mm. This process continues until $l = 0.2$ mm, that is, the probe is 0.2 mm inside the second material (see Fig. 1(a)). This results in a total of 900 simulations per scenario. After each simulation, the permittivity is extracted from the reflection coefficient using the method described in Section II-A. The simulations are carried out for both the thin and the thick probes, at a frequency of 1 GHz.

Two sets of simulations are performed. The first set show the measured relative permittivity of four different media as a function of the probe's distance to the separation layer. These media are Medium 1: $\epsilon_{r1}=10$ (layer 1) and $\epsilon_{r2} = 100$ (layer 2), Medium 2: $\epsilon_{r1} = 30$ and $\epsilon_{r2} = 50$, Medium 3: $\epsilon_{r1} = 70$ and $\epsilon_{r2} = 50$, and Medium 4: $\epsilon_{r1} = 100$ and $\epsilon_{r2} = 10$. The conductivity of all 4 media is $\sigma = 5$ S/m. The results summarized in Fig. 3 show that the measured relative permittivity is equal to ϵ_{r1} when the probe's tip is in the first layer at a relatively large distance from the second layer. As the probe advances toward the second layer the extracted permittivity changes at a rate that strongly depends on the permittivity contrast c of the layers, that is $c = |\epsilon_{r1} - \epsilon_{r2}|/|\epsilon_{r1} + \epsilon_{r2}|$.

As per the definition provided earlier, and taking $\lambda = (3\epsilon)^{-1} \approx 0.1$, the PD is the value of l_1 where the observed permittivity changes by $\pm 10\%$ compared to ϵ_{r1} . The measured PD is indicated in Fig. 3 by the dashed lines. The PD values for media 1 to 4 are, respectively, 0.06 mm, 0.2 mm, 0.14 mm, and 0.08 mm. The largest the PD is observed in Medium 1, which has the highest contrast and the lowest initial permittivity. While the change rate of Media 1 and 4 are very close to one another because of the lower initial permittivity value, the PD is detected at a higher distance in medium 1 (0.6 mm). As expected, the minimum the PD is observed in Medium 3, which has the lowest contrast of all. The second set of simulations shows the penetration depth for different combinations of permittivity values assigned to each layer, ranging from 10 to 100 with increments of 10. The distance of the probe to the separation layer is from $-0.7 \leq l_1 \leq 0.2$ mm with steps of 0.01 mm. This amounts to 9,000 simulations. Each simulation is performed for the tick and thin probes, at 1 and 5 GHz, considering $\sigma = 5$ S/m and $\sigma = 10$ S/m, totalling 54,000 simulations.

A. Influence of the layer's permittivity of the PD

Fig. 4(a) shows the resulting PD of the thin probe at 1 GHz determined from the 9,000 simulations. The conductivity of both layers is $\sigma = 5$ S/m. As it can be seen, when the first

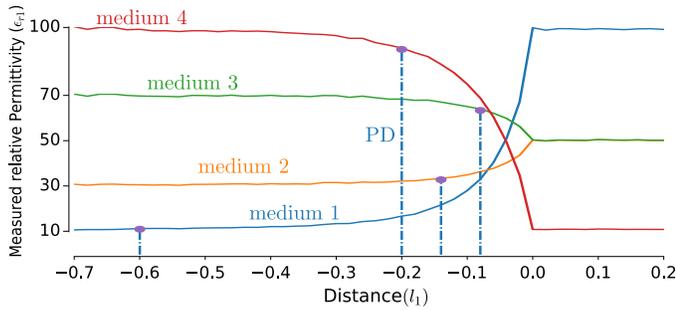


Fig. 3. Measured relative permittivity at different distances (l_1) for different layers contrasts.

and second layer's permittivities are the same, the PD is zero (diagonal line in the graph), and the PD increases with the contrast between the layers. The highest PD is 0.6 mm and is again observed in the medium having the highest analyzed permittivity contrast, with $\epsilon_{r1} < \epsilon_{r2}$, that is $\epsilon_{r1} = 10$, and $\epsilon_{r2} = 100$.

B. Influence of the layer's conductivity of the PD

The same set of simulations with the relative permittivity ranging from 10 to 100 is performed considering $\sigma_1 = 5$ S/m and $\sigma_2 = 10$ S/m. The obtained the PD for the thin probe is shown in 4(b). The results show the same pattern as observed in the case where the layers have different permittivities. However, the PD is generally lower than what is detected in the first set of simulations. This can be explained by the fact that the absorption of an electromagnetic field in a material is directly proportional to its conductivity. As a result, the contribution of the second layer on the reflection coefficient is reduced as its conductivity increases. This lowers the PD, resulting in the highest penetration depth to be 0.4 mm for this test case.

C. Influence of the probe diameter on the PD

Previous works suggest that the PD increases with the probe diameter [8], [12] regardless of the definition of the PD. The same result is also confirmed in this work. Fig. 4(c) shows the PD of the thick probe at 1 GHz for the same relative permittivity contrast as Fig. 4(a). Comparing the values of Fig. 4(a) and 4(c) shows that for the same contrast, the probe with the larger diameter has a higher the PD. The highest the PD (2 mm) is observed with the thick probe for $\epsilon_{r1} = 10$, and $\epsilon_{r2} = 100$, which is about 3 times higher than the PD observed with the thin probe. This observation is in agreement with earlier work that examined the impact of probe diameter on the PD of open-ended coaxial probes [3], [8].

D. Influence of the frequency on the PD

Whereas the effect of diameter on the PD is consistent across the literature, this is not the case for frequency. Simulations and experimental results of different works regarding the effect of frequency on the PD contradict each other [8], [9]. Thus, the same three sets of simulations performed earlier are repeated at 5 GHz. The results shown in Fig. 4(d)-(f) indicate

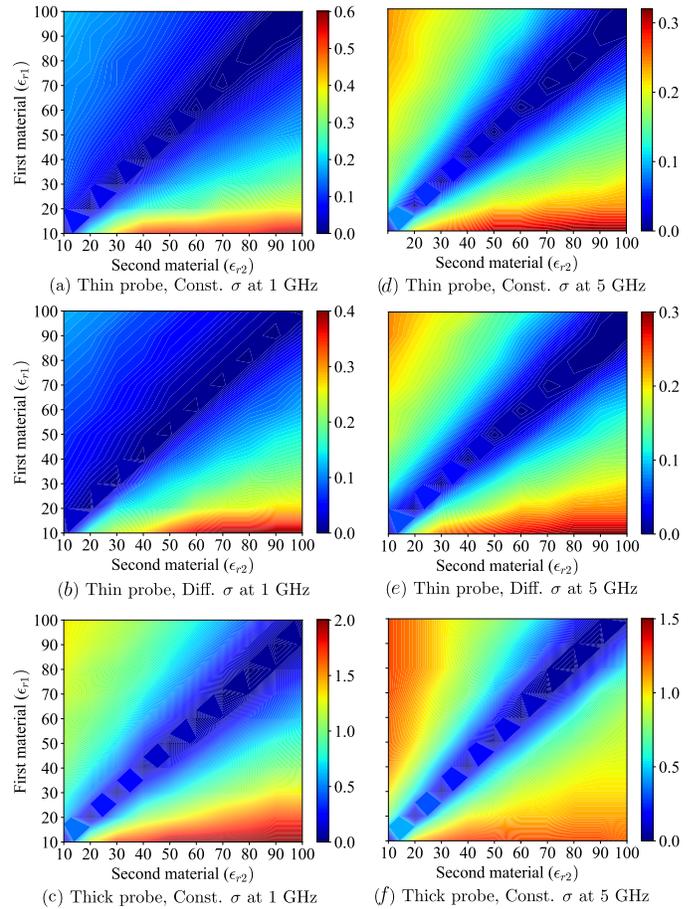


Fig. 4. Penetration depth [mm] (colour scale) for different scenarios.

that the PD decreases with frequency as compared to what is seen in Fig. 4 (a)-(c). This can be explained by the fact that wave dissipation in a material increases with frequency, causing the penetration depth of an electromagnetic wave to be inversely proportional to its frequency.

IV. CONCLUSION

This paper provides new insights into the effect of various tissue parameters on the PD in dielectric spectroscopy of a two-layer medium. The definition of the PD is based on the change in permittivity. The results drawn from 54,000 simulations show that the tissue layer's characteristics have a strong impact on the accuracy of spectroscopy measurements. In fact, the PD increases as the difference in permittivity between the tissue layers increases, but the PD decreases as the conductivity of the second layer increases. Additionally, the results show that using a smaller diameter probe can result in more precise permittivity measurements. Finally, the results indicate that the PD decreases with frequency since wave dissipation in the tissue increases with it. The latter point helps clarify contradictory claims previously reported in the literature regarding the influence of frequency on the PD.

The findings reported in this paper can greatly aid in quantifying and understanding the accuracy and sensitivity of

biological tissue classification using dielectric spectroscopy and, in the near future, will guide the design of specialized dielectric spectroscopy probes for in-vivo measurements.

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